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Marx and the Other Sraffa: The Insignificant Empirical Effect of Price-Value Deviations on Economic Aggregates

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1   The Visible Sraffa[1]

Sraffa's elegant and elliptical book transformed the treatment of prices of production. His stated agenda was to ground a critique of some central propositions of neoclassical theory, particularly its reliance on the notion of an aggregate production function whose marginal products of labor and "capital" determines wage and profit rates, respectively (Sraffa 1960, pp. v–vii). From the neoclassical perspective, "[w]hat a social class gets is, under natural law, what it contributes to the general output of society" (Clark 1891).

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[1] I wish to express my gratitude to Riccardo Bellofiore and Scott Carter for their illuminating work on Sraffa's archives. My use of the phrase "the Other Sraffa" is a reference to the conference 'The Other Sraffa: Surprises in Archive?' at the University of Bergamo in December 2010 and to the subsequent volume (Bellofiore and Carter 2014).

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Robinson (1953) pointed out that the quantity of "capital" was itself the aggregate price of a heterogeneous bundle of capital goods. Since individual prices of production depend on the profit rate, the aggregate price of capital goods could not also determine the profit rate. Sraffa provided the formal foundation for this argument, which in turn became a central focus of the famous Cambridge Capital Controversies (Harcourt 1969; Cohen and Harcourt 2003).

Sraffa's impress led to many neo-Ricardian representations of Marx. Marx's great emphasis on the length of the working day as the proximate source of a surplus product in pre-capitalist societies, and of surplus value in capitalist societies, disappears from view under the assumption that the number of workers was a proxy for their total labor time (Bellofiore 2014, pp. 207, 214). This Ricardian element is present in Sraffa's own formulation, in which a surplus product is represented as coming into existence due to an advance in technology (Sraffa 1960, pp. 6–7). It is a particularly curious formulation given the well-known history of working class struggles about conditions of work, and the fact that even today the length and intensity of the working day vary enormously between countries. In most advanced capitalist countries, through a long struggle, workers have achieved a 40-hr working week at a moderate pace. In China and some other developing countries, a common practice is "996": 9am to 9pm, six days a week (72-hrs) at a break-neck pace. This is a non-trivial point, since comparisons among "techniques" would then have to account for variations in the conditions of labor extraction (Shaikh 2016, pp. 206–220).[2]

A common neo-Ricardian claim is that Marx's "erroneous transformation algorithm" arose because he "did not have the method of simultaneous equations at his disposal" (Gehrke and Kurz 2006, p. 214). Yet in Marx's presentation of his algorithm, he is careful to state that it is a *first* step because any deviations of prices from values implies that costs previously expressed in labor value terms must be further adjusted to reflect the new prices.

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[2] A longer and or more intense working day would count as a lower labor coefficient and hence as a lower unit labor value. Since Sraffa prices equal labor values at $r = 0$, this would lower their absolute levels.

We had originally assumed that the cost-price of a commodity equaled the value of the commodities consumed in its production. But for the buyer the price of production of a specific commodity is its cost-price, and may thus pass as cost-price into the prices of other commodities. Since the price of production may differ from the labor value of a commodity, it follows that the cost-price of a commodity containing this price of production of another commodity may also stand above or below that portion of its total value derived from the value of the means of production consumed. It is necessary to remember this modified significance of the cost-price, and to bear in mind that there is always the possibility of an error if the cost-price of a commodity in any particular sphere is identified with the value of the means of production consumed by it. Our present analysis does not necessitate a closer examination of this point. (Marx 1967b, Chapter IX, pp. 164–165)

In other words, his derivation is the beginning of an *iterative* process. Iteration has been around since Gauss (1823), and even today is commonly utilized in computer calculations. The steps involved could easily have been implemented by Marx, and have been shown to lead to the standard simultaneous equation solution (Shaikh 1977; Morishima and Catephores 1978).

Another often-mentioned point in that literature is that, for any common total output price (i.e. for any common price-level), the deviations of prices from values can yield aggregate profits different from aggregate surplus value.[3] In the algebraic domain, this is touted as the definitive criticism of Marx. Yet the very *same* problem arises when we consider prices (market, monopoly, etc.) different from Sraffian prices: aggregate profits will then differ from aggregate competitive profits. It follows that all models of pricing have a "transformation problem". Once again, it is most curious that neo-Ricardian followers of Sraffa have overlooked this obvious point. I have argued that in all cases this difference is the net result of balance-of-payments transfers between the capital and

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[3] Sraffa shows that this problem arises solely from the difference between actual <u>output proportions</u> and standard ones. In the latter case, all aggregates have the same internal proportions (i.e. they are the same vector except for scale), so that any sets of prices will yield the same ratios. Hence keeping the value of money constant (keeping the price production of aggregate output equal to its value) will make all other price aggregates equal to their value counterparts.

non-capital circuits on which Marx himself places great emphasis (Shaikh 2016, Ch. 6).

My present concern relates to the preceding point along a different dimension: at an empirical level, how relevant is the putative complexity of prices of production, and how important are the effects of price-value deviations on economy-wide aggregates? It turns of that the Other Sraffa, brought out from his archive lair, has a lot to say on this.

2   The Other Sraffa

The material in Sraffa's archives, now publicly available thanks to the efforts of a dedicated band of scholars such as John Eatwell, Jonathan Smith, Giancarlo de Vivo, Murray Milgate, and Scott Carter and Riccardo Bellofiore, came as a shock to some of Sraffa's followers, particularly those whose comments about Marx and the labor theory of value (LTV) had been critical and dismissive. Yet according to "anecdotal evidence put forward by his friends and colleagues, … [Sraffa] was always an adherent of the LTV" (Bellofiore 2014, p. 205), the "role of Marx in the construction of Sraffa's 1960 book turns out to be more profound than could previously have been guessed before the opening of the Sraffa Archive, and Sraffa's reference to Marx's value theory persists even after the publication of the book" (Bellofiore and Carter 2014, p. 19).

In the late 1920s, Sraffa was critical of the labor theory of value as he understood it. By 1927 he had "discovered the Quesnay–Smith–Ricardo 'surplus approach' whose ultimate result was to be a restatement of Marx" and by the 1940s had read Marx's *Theories of Surplus Value* and *Capital* Volume II and obviously Volume III containing Marx's transformation from labor values to prices of production. In the early 1940s, as he was getting back to work on his book, "Sraffa was convinced that his study would have shown that Marx was unequivocally correct." (Bellofiore 2014, pp. 210–212). Commenting on Bortkiewicz's simultaneous equation solution for prices of production, Sraffa say: "[T]he real objection (though somewhat vaguer) is this: that B[ortkiewicz]'s point of view, for the sake of obtaining absolute exactness *in a comparatively trifling matter*, sacrifice (by concealing it) the essential nature of the question—that is,

that commodities are produced *by labour* out of commodities. (D1/91/16; emphasis added)". We also know that Sraffa underlined a passage in T.A. Jackson's *Dialectics: The Logic of Marxism*, which says "Capitalist production is therefore a system of producing commodities *from* commodities (raw materials, machinery etc.) *by means* of commodities (*the labour power* of wage labourers)" (Bellofiore and Carter 2014, p. 214, emphasis added). This is a direct reference to Marx's central points that labor-power becomes a commodity under capitalism. It provides a different perspective on the title of Sraffa's *The Production of Commodities by Means of Commodities*.

Marx did not live to write Volumes II and III of *Capital*. Engels compiled the material in those books, after of a long and arduous effort, from a mass of notes that Marx left behind. Vol III of *Capital* appeared more than a decade after Marx's death (Marx 1967b, Prefaces, 1967a). To this day, we do not know much about the excluded material. With this in mind, there is a striking correspondence in Marx's (incomplete) published material between his notion of a "standard" industry whose price of production need not deviate from its value and Sraffa's own standard industry with exactly the same property. Marx first establishes that the formation of a common rate of profit implies that the prices of production of individual industries will deviate from their labor values. As previously noted, he goes on to say that these deviations must be fed back onto costs previously measured in terms of labor values. But, one particular industry will be exempt from price-value deviations because it has a special "structure" with a particular composition: "The capital invested in some spheres of production has a *mean, or average, composition*, that is, it has the same, or almost the same composition as the average social capital … In these spheres the price of production is exactly or almost the same as the value of the produced commodity expressed in money. *If there were no other way of reaching a mathematical limit, this would be the one"*. This average composition is an "ideal average, i.e., an average that does not really exist, i.e., a[n] *ideal as a standard* …In the case of [such capital] … the price of production is … the same or almost the same as the value, and the profit the same as the surplus value produced by them … with the same, or approximately the same *structure* " (Marx 1967b, Ch X, pp. 173–174, emphasis added).

So the industry of a mean, or average composition, "the ideal as standard", has a composition that is different from the "social average". Yet Marx goes on to say that sum of "the profit on the social average capital is equal to the sum of surplus value", which does not follow (op. cit.). We do not know whether in his unpublished notes he meant this as an exact statement or as an empirically valid statement as Sraffa insists. However, we can say that just as the profit rate and aggregate profit corresponding to prices of production theoretically differ from the value rate of profit and surplus value, respectively, so too will Sraffian total profit and the profit rate differ from those measured at market, monopoly, prices. Those who insist that price-value deviations invalidate Marx's theory of surplus value should at least have the grace to admit, under their own logic, that there is a *universal* transformation problem (Shaikh 2016, pp. 217–226).

Sraffa also begins from a situation in which individual prices equal labor values. In Marx's case this is because he chooses labor values as his starting point to establish that the creation of profit (Vol I) is independent of transfers of any sort (Shaikh 2016, pp. 208–212). In Sraffa's case it is because he starts at a point at which the wage share happens absorb all net national product per worker (hr.), in which case the profit share and profit rate are zero and his prices equal labor values. He next considers a lower wage share, so that a positive rate of profit arises. Then prices cannot remain equal to values because in "deficit" industries with high capital-labor ratios the saving in wage costs would not be sufficient to generate a normal rate of profit, while in "surplus" industries with low capital-labor ratios the same effect would be more than sufficient. But, "[t]here would be a "critical proportion of labour to means of production … which marked the watershed between 'deficit" and 'surplus' industries", and this critical proportion would fall somewhere between the lowest and highest capital-labor ratios. In the elegant argument that follows, Sraffa establishes that one can construct a composite sector whose price would remain equal to its labor value (its initial price) regardless of the level of the rate of profit, and whose output-capital ratio evaluated at prices of production remain equal to its labor value equivalent. This *standard* industry is ideal in two senses: because it does not exist as such; and because it serves as the fulcrum for the analysis of changes in the wage (profit) relation.

By construction, the standard output-capital ratio is invariant to changes in distribution. When wages are zero and all net output goes to profit, the common rate of profit is at its maximum. Then all industries, including the standard one, will have the same output-capital ratio. Since the output-capital ratio of the standard industry is invariant, it is also the maximum rate of profit. As a result, the wage share in the standard sector would be linearly related to the actual profit rate. In the actual system the output-capital ratio would not be invariant and the relation between wage share and the profit rate would not be a straight line (Sraffa 1960, pp. 12–23).

In the 1940s, Sraffa had hypothesized that the aggregate output-capital ratio was indeed constant, and hence equal to the maximum rate of profit. He "thought that his inquiry would vindicate the 'Old Moor'". Even after his 1960 book in which he had shown that his "Hypo" was not exactly true in theory, he "maintained a positive judgment on Marx's transformation procedure and …[viewed the] results of his book in terms of capitalist exploitation within an amended Marxian discourse" (Bellofiore 2014, p. 199).

Sraffa's "Hypo" drove his research for several years because it would have provided a simple relation between the wage share and a uniform rate of profit at *any* given output proportions, that is, in the actual economy. Let $Y(r)$, $K(r)$ represent actual net output (net national product) and capital stock in prices $p(r)$ calculated at some rate of profit (r), L represent total hours of labor, and ω, r represent equalized money wages and profit rates. Then ωL is the aggregate wage bill, $rK(r)$ is aggregate profit, and from the national accounting identity $Y(r) \equiv \omega L + rK(r)$.

Denoting the wage share as $w(r) \equiv \dfrac{wL}{Y(r)}$ and the output-capital ratio as $R'(r) = \dfrac{Y(r)}{K(r)}$ gives a relation between the two at any given rate of profit for arbitrary output proportions:

$$w(r) = 1 - \frac{r}{R'(r)} \tag{16.1}$$

If $\acute{R}(r)$ was constant at all rates of profit, in keeping with Sraffa's Hypo, then it would have to be equal to the maximum rate of profit R corresponding to a zero wage share. In this case, the actual wage share would be a linear function of the rate of profit. Conversely, if $\acute{R}(r)$ did not vary much, which we will see is the empirical case, this would explain the virtually universal finding that actual wage-profit curves were near-linear.

Sraffa's Hypo was originally "based on the "statistical compensation of large numbers" (D3/12/35: 28)" (Perri 2014, p. 109). Indeed, in his unpublished notes Sraffa himself says that

> the propositions of M[arx] are based on the assumption that the composition of any large aggregate of commodities (wages, profits, constant capital) consists of a random selection, so that the ratio between their aggregates (rate of surplus value, rate of profit) is approximately the same whether measured at 'values' or at the prices of production corresponding to any rate of surplus value… . This is obviously true.

In response to Eaton's (1960) review, Sraffa defends Marx again: "It is clear that M[arx]'s pro[position]s … are based on the assumption (*justified in general*) that the aggregates are of some average composition. This is in general *justified in fact*, and since it is not intended to be applied to detailed minute differences it is all right … This should be good enough till *the tiresome objector arises*. If then one must define which is the average to which the comp[osition] should conform for the result to be *exact* and not only approximate, it is the S[tandard] Comm[odity]" (Bellofiore 2014, pp. 220–221, emphasis added).

3   Empirical Investigations

Although it is often said that economics is too much like physics, to a physicist economics is not at all like physics. The difference is in the scientific method of the two fields: theoretical economics uses a top-down approach in which hypothesis and mathematical rigor come first and empirical confirmation comes second. Physics, in contrast, embraces the bottom up 'experimental philosophy' of Newton, in which 'hypotheses are

inferred from phenomena, and afterward rendered general by induction' …
if economics were to truly make empirical-verification the ultimate arbiter
of theories … [this] would force it to open up to alternative approaches.
(abstract)[4]

Early Sraffian literature concentrated on elaborations of Sraffa's math-
ematics (Steedman 1977; Schefold 1989). Over time a portion of the
literature shifted to the empirical investigation of actual relative prices of
production and of actual wage-profit frontiers (Krelle 1977; Ochoa 1989;
Shaikh 2012, 2016, Ch. 9; Bienenfeld 1988; Chilcote 1997; Mariolis
and Soklis 2010; Mariolis and Tsoulfidis 2011; Iliadi et al. 2014; Shaikh
1984). This growing body of evidence showed that wage-profit curves
and even individual price curves are near-linear. Han and Schefold (2006)
found that in actual European tables reswitching and reverse capital
deepening were rare. On the other side, it has been shown that such
results do not reinstate the neoclassical aggregate production function
that was Sraffa's own focus (Shaikh 2016, pp. 431–433, 440). In this
regard, Zambelli's two papers (2004, 2018) are particularly innovative. In
the first paper he generates artificial economies consisting of different
numbers of commodities with many different viable methods of produc-
tion (blueprints) for each and finds that only about 40 percent are even
look like aggregate production functions. In the second paper, he uses
actual production methods from 30 countries over 15 years to create a
vast set of potential economies. Here his conclusion is more draconian:
as-if aggregate production functions are never neoclassical over the whole
domain, isoquants are never neoclassical, and the capital-labor ratio is
seldom appropriately (negatively) sloped.

I wish to take up a different question: In the context of actual systems,
is Sraffa correct to say that statistical considerations validate Marx's notion
that price aggregates are essentially equivalent to corresponding labor
value aggregates?

Actual US input-output tables for 1947, 1958, 1963, 1967, 1972 (71-
order) and 1998 (65-order)[5] were used to calculate vectors of

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[4] J. Doyne Farmer is Professor of Mathematics at Oxford University where he is Director of the
Complexity Economics at the Institute for New Economic Thinking at the Oxford Martin School.
[5] Sources and methods for the 1947–1972 data are in Shaikh (1998) Appendix 15.2, and for 1998
are in Shaikh (2012) Data Appendix. In order to isolate the effects of changes in *relative* prices on

intermediate input, workers consumption, surplus product, net output and capital stock.[6] Each of these was measured at prices of production corresponding to the *observed* rate of profit (displayed in the first line in each table) in that year and in direct prices (prices proportional to labor values). The *ratios* of the aggregate production price of each bundle to its direct price are called ConstCap, VarCap, SurplusProd, NetOutput, SV (Surplus Product/Variable Capita), rop (rate of profit = Surplus Product/ Capital), and R1 (Output/Capital), respectively. Each ratio was calculated for the circulating capital case in which the capital stock is equal to ConstCap and the fixed capital case in which the capital stock is fixed capital. Since Sraffa prices equal labor values at a zero rate of profit ($r = 0$), *the preceding ratios also represent the effect on each variable of moving from $r = 0$ to $r = robserved$*. In the circulating capital case, average price-value ratios of various aggregates range between 0.95 and 1.03 (Table 16.1), while in the fixed capital case they range between .097 and 1.05 (Table 16.2). As Sraffa had predicted, *in actual conditions there is no*

Table 16.1 Ratios of price and value aggregates at observed rates of profit and profit share: Circulating capital

Matrix size	71	71	71	71	71	65	
	1947	1958	1963	1967	1972	1998	*Average*
Observed rate of profit (robserved)	0.32	0.32	0.36	0.40	0.36	0.23	0.33
Observed profit share (robserved/R)	0.32	0.30	0.35	0.39	0.35	0.22	0.32
ConstCap	1.02	1.02	1.03	1.04	1.03	1.02	*1.03*
VarCap	0.99	0.98	0.97	0.97	0.96	0.99	*0.98*
SurplusProd	0.98	0.98	0.97	0.97	0.98	0.98	*0.98*
NetOutput	1.00	0.98	0.97	0.97	0.97	0.98	*0.98*
SV	1.03	1.00	1.00	1.00	1.02	0.99	*1.01*
rop	0.97	0.96	0.94	0.94	0.95	0.96	*0.95*
R1	0.97	0.96	0.94	0.94	0.94	0.96	*0.95*

various aggregate bundles, we must keep the price *level* constant. Total market price is defined as the market value of gross output, and total direct price and total price of production at any given profit rate are scaled to match total market price. This keeps the value of money the same across all three types of prices.

[6] US data has several advantages. The data is good, the time span is long, US economy is relatively closed (from 1977–2007 almost 90 percent of goods are produced domestically), and there are good domestic approximations to the input-output structure of imported commodities.

Table 16.2 Ratios of price and value aggregates at observed rates of profit and profit share: Fixed capital

Matrix sze	71	71	71	71	71	65	
	1947	1958	1963	1967	1972	1998	*Average*
robserved	*0.24*	*0.18*	*0.21*	*0.23*	*0.19*	*0.11*	*0.19*
Observed profit share (robserved/R)	*0.29*	*0.26*	*0.28*	*0.31*	*0.28*	*0.27*	*0.28*
ConstCap	1.02	1.03	1.04	1.04	1.03	1.03	*1.03*
VarCap	0.98	0.99	0.98	0.99	0.98	1.01	*0.99*
SurplusProd	0.98	0.96	0.95	0.94	0.96	0.98	*0.96*
NetOutput	0.98	0.97	0.97	0.97	0.97	0.96	*0.97*
SV	0.99	0.97	0.97	0.96	0.97	1.00	*0.98*
rop	1.04	1.04	1.05	1.05	1.04	0.98	*1.04*
R1	1.04	1.06	1.07	1.08	1.06	1.00	*1.05*

Table 16.3 Ratios of price and value aggregates in observable range r/R = 0.2 to 0.4, circulating capital

	1947	1958	1963	1967	1972	1998	*Average*
ConstCap	0.99	1.04	0.95	1.02	1.02	1.02	*1.00*
VarCap	1.01	1.03	1.01	0.98	0.98	0.99	*1.00*
SurplusProd	1.01	1.01	1.02	0.98	0.99	0.98	*1.00*
NetOutput	1.01	1.01	1.02	0.98	0.98	0.98	*1.00*
SV	1.01	1.00	1.01	1.00	1.01	0.99	*1.00*
rop	1.02	1.05	1.04	0.97	0.97	0.96	*1.00*
R1	1.02	1.03	0.97	0.97	0.97	0.97	*0.99*

*effective difference between aggregate*s. The last line in Tables 16.1 and 16.2 represent the price-value ratio of net output-capital that was Sraffa's singular focus: over 1948–1998, in the circulating capital case the ratios range from 1.00 to 1.08; in the fixed capital case they range from 0.97 to 1.03, with an average of 0.99.

Across 29 advanced countries (27 European countries plus the USA and Canada), the observed profit share r/R varies between 0.25 and 0.32 (Shaikh 2016, T. 9.5, p. 388). Taking the range of r/R = 0.20–0.40 as feasible outer bounds of potential profit shares, I examine how much each ratio changes between the lower and upper bounds. Tables 16.3 and 16.4 show there is effectively no difference over the socially feasible range of the profit share: in the circulating capital case, the final and beginning

Table 16.4 Ratios of price and value aggregates in observable range r/R = 0.2 to 0.4, fixed capital

Ratios between r/R = 0.20 and r/R = 0.40	1947	1958	1963	1967	1972	1998	*Average*
ConstCap	1.01	1.02	1.02	1.02	1.02	1.02	*1.02*
VarCap	0.99	0.99	0.99	0.99	0.99	1.00	*0.99*
SurplusProd	0.98	0.97	0.97	0.97	0.97	0.96	*0.97*
NetOutput	0.99	0.98	0.98	0.98	0.98	0.98	*0.98*
SV	0.99	0.98	0.98	1.01	0.98	1.02	*0.99*
rop	1.03	1.03	1.04	1.03	1.03	0.98	*1.02*
R1	1.03	1.04	1.03	1.05	1.04	0.99	*1.03*

ratios are essentially the same, while in the fixed capital case they range between 0.97 and 1.03.

Next, we consider all the preceding variables over the full range of r/R from 0 to 1. Each variable is normalized to its corresponding labor value, and since Sraffa prices equal labor values at r/R = 0, each ratio begins at 1 and spreads out as r/R increases. In theory, the paths could be complicated. However, as shown in Fig. 16.1, in all years almost all the empirical paths of the variables are near-linear, with only a few have some smooth curvature. Most ratios vary less than ten percent over their whole level, with only a few around fifteen percent.

As previously noted, in the 1940s Sraffa had hypothesized that the capital ratio would be constant over the range. He subsequently showed that it was not. At r/R = 0, Sraffa prices are equal to labor values, so all initial output-capital ratios represent the ratio of the labor value of net output (living labor hours) to the labor value of the capital stock. These will be different according to the industry involved. Then as the profit share varies, all output-capital ratios approach the same ratio R, which is the output-capital ratio of the standard industry (Sraffa 1960, pp. 16–17). Figure 16.1 had already established the smoothness and modest non-linearity of these paths. Table 16.5 shows that the average variation of the output-capital ratio in the circulating capital case is 0.86, while that of the fixed capital case it is 1.17. This is for the full, socially impossible range of the profit share, with workers living on air at one end and capital euthanized at the other. Tables 16.3 and 16.4 had already demonstrated that within a socially feasible range the output-capital ratio hardly varies:
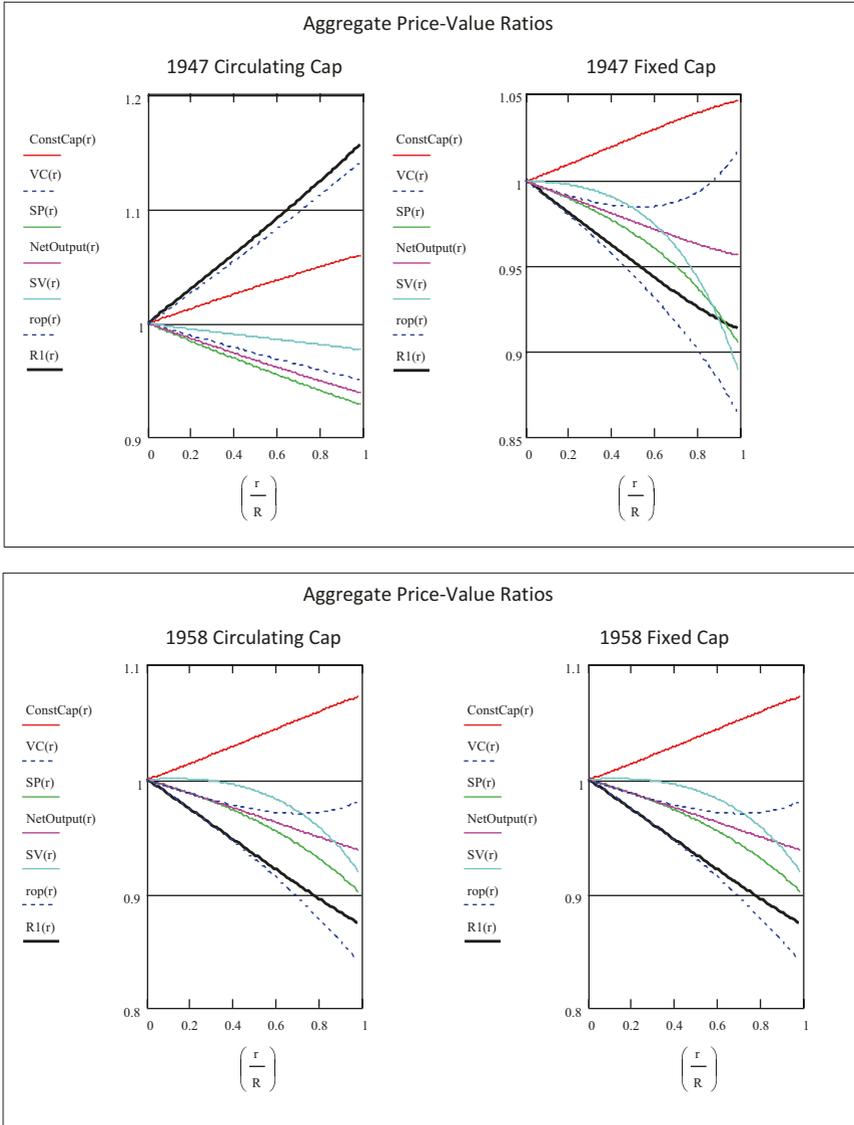
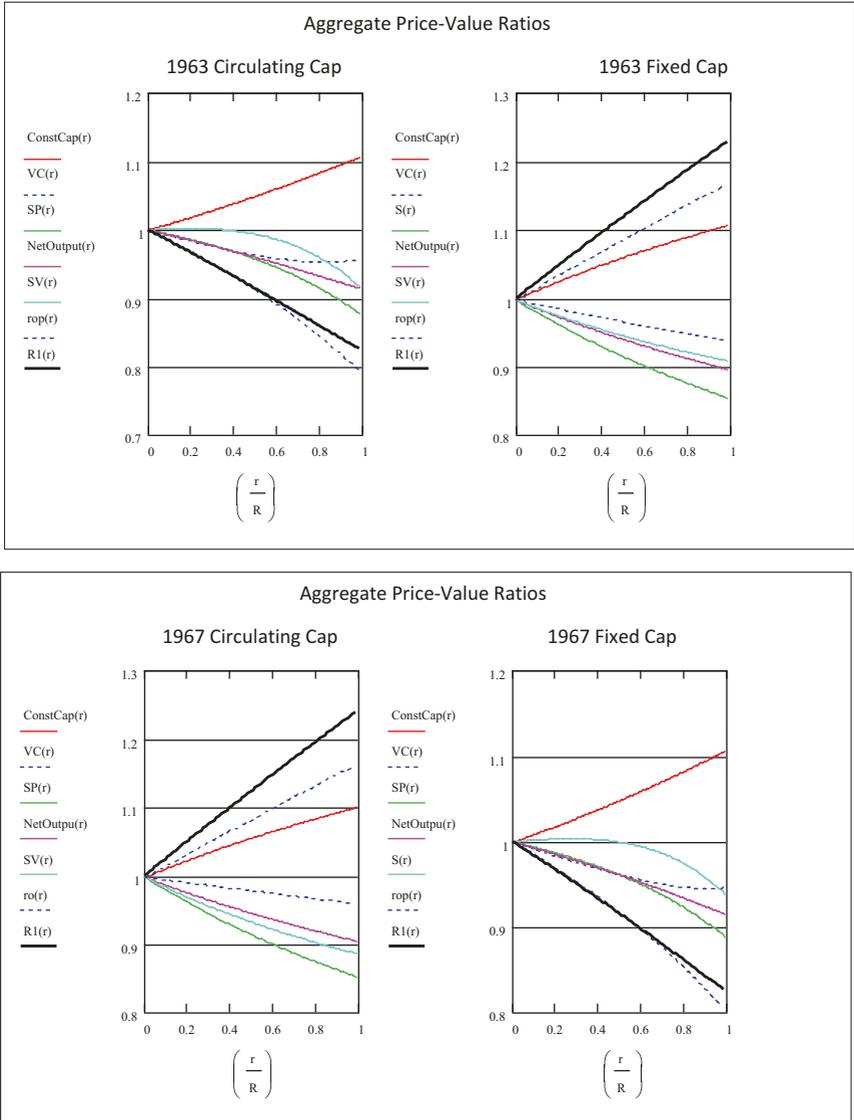Fig. 16.1   Aggregate price-value ratios
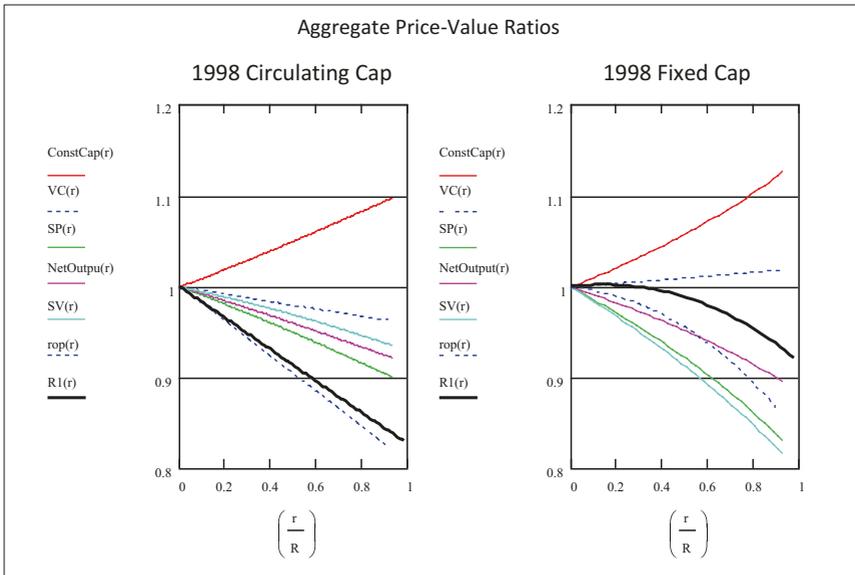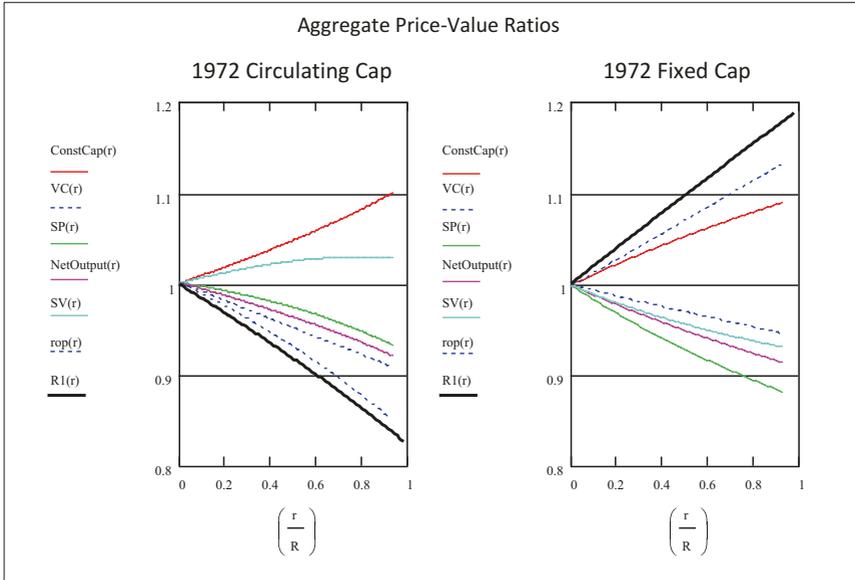
Fig. 16.1 (continued)

Fig. 16.1 (continued)

Table 16.5 Sraffa's hypo: Output-capital ratio over full range (r/R = 0 to 1)

	1947	1958	1963	1967	1972	1998	*Average*
Circulating capital	0.91	0.92	0.86	0.83	0.83	0.83	*0.86*
Fixed capital	1.15	1.21	1.29	1.24	1.19	0.92	*1.17*

in advanced capitalist countries, Sraffa's Hypo is completely correct. This in no way detracts from the beauty of his subsequent formulation. It just says that theory must always be tempered by the actual properties of its object of investigation—precisely as Sraffa does.

Finally, I would like to affirm my debt to my student Eduardo Ochoa for his pathbreaking PhD dissertation and subsequent publication (Ochoa 1984, 1989). The input-output tables and related variables for 1947–1972 that I have utilized were his constructions. He begins with a classical fixed capital model of prices of production with a given wage basket of commodities and corresponding sectoral and average rate of profit in each year. He calculates the Marxian rate of surplus value, the labor value profit rate, and direct and vertically integrated ratios of fixed capital to living labor. He finds a high degree of correspondence between direct prices (prices proportional to labor values), market prices and prices of production (the latter calculated at the observed rate of profit). The rates of profit measured in these different prices are very close. In retrospect, this further evidence confirming the Other Sraffa's sense about the empirical validity of Marx's approach. Finally, Ochoa turns to a fixed capital model of Sraffa prices over their full range finds wage-profit curves to be generally smooth and near-linear—suggesting that reswitching would be a rare empirical phenomenon in US data. This is despite the fact that the composition of the actual output vector is very different from that of Sraffa's standard commodity (Ochoa 1984, Chs. V–VII).

4   Summary and Conclusions

The 1960s Sraffa is right to say that the theoretical output-capital ratio will vary with the profit share r/R. And the Other Sraffa is right to say that this variation is insignificant at an empirical level, which validate Marx's intuition.

The Sraffian literature originally concentrated on elaborations of Sraffa's mathematics emphasizing "wiggly" prices and wage-profit curves. As work shifted to empirical investigation, a growing body of evidence found wage-profit curves and even individual price curves are near-linear, so that reswitching and reverse capital deepening were rare at the observed empirical level. At the same time, it was emphasized that these results do not reinstate the neoclassical aggregate production since the price of production of capital and net output itself depended on the rate of profit.

At a more detailed level, at the observed rate of profit, the price-value ratios of intermediate inputs, the wage basket, the surplus product, net product, profit-wage ratio, rate of profit, and output-capital were close to 1 in both circulating and fixed capital models. The price-value ratio of net output-capital that was the Sraffa's singular focus: over 1948–1998, in the circulating capital case the ratios range from 1.00 to 1.08; in the fixed capital case they range from 0.97 to 1.03, with an average of 0.99. Further, since the observed profit share ranges in 29 advanced countries between 0.25 and 0.32, $r/R = 0.20$–0.40 was taken as feasible outer bounds. Once again, there is effectively no difference the output-capital ratio over the socially potential range.

The last step was to consider all the preceding variables over the full range of r/R from 0 to 1. In theory, the paths could be complicated. However, in all years almost all the empirical paths of all variables were near-linear, with only a few have some smooth curvature. Most ratios vary less than ten percent over their whole level, with only a few around fifteen percent. Between the two full limits, in the circulating capital case the average variation of the output-capital ratio is 0.86, while in the fixed capital case it is 1.17. These are algebraic limits of course. Within any feasible range the output-capital ratio hardly varies. This reminds us theory must always be concerned by the actual properties of its object of investigation. This is the greatest beauty of Sraffa's contribution.

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