Abstract

This paper examines three linked propositions: Sraffa's insistence in his archival notes that Marx was "of course" right in thinking that price and labor value aggregates are essentially the same at the empirical level. Sraffa's turns out to be entirely correct: Marxian labor value aggregates and their price equivalents exhibit very small deviations, generally less than five percent. I would argue that any representative small models should reflect such properties. Second, Pasinetti's is correct to caution that actual price-paths are likely to be quite a bit simpler than those portrayed in the theoretical literature. This is fully confirmed by the 403 industry price curves generated from 2002 matrix. Third, Marx's transformation procedure is actually an iterative procedure that provides a powerful path to an empirically sufficient general approximation of Sraffa prices. Bienenfeld derives powerful first and second iterates, each of which starts and ends at the same points as the corresponding individual price curve. The second iterate provides excellent fits for the 403 price curves in this study. I would argue that this simple nonlinear expression constitutes an empirically sufficient general form of Sraffa prices.

Keywords: Sraffa, Marx, prices, labor values, economic aggregates, stochastic effects

An Empirically Sufficient Form for Sraffa Prices

1. Introduction

I offer this an essay in honor of my teacher and mentor Luigi Pasinetti. I entered graduate school at Columbia University in 1967. The goal was to study economics, to understand how economies and peoples function at national and international levels. In this time of political turbulence in the US, with civil rights, feminist, and anti-war movements roiling across the landscape, the representations of neoclassical theory seemed bizarre. Equally unsatisfactory was opposite tendency to rely on "imperfections" to explain the world as we found it. There are no imperfections without perfections. As did so many others at that time, I explored anthropology and politics. I began to understand the difference between abstraction as a starting point and idealization as a goal. In this way I began to grasp the distance between the classical notion of real competition and the neoclassical representation of perfect competition.

It was in that context that I first encountered Harcourt's remarkable article on the Cambridge Capital Controversy. I discovered that there was a community of scholars such as Sraffa, Joan Robinson, Pasinetti, Garegnani, Bhaduri, Schefold, and so many others, who had long been following another path. This in turn led me back to Smith, Ricardo and Marx, to Dobb and Meek. Then Luigi Pasinetti appeared at my university in 1971, as if by magic, to give a course on Sraffa. I took assiduous notes in every lecture, and during the week I rewrote them from scratch. In the end, I had full a text of Pasinetti's lecture material, some of which later appeared in his *Lectures on The Theory of Production*. His calm and precise lectures, drawing upon Sraffa's elegant and mysterious text, became my touchstones.

In this paper I examine three relatively under-appreciated relations linking Marx, Sraffa and Pasinetti: Sraffa's insistence in his archival notes that Marx was "of course" right in thinking that price and labor value aggregates would essentially the same; Pasinetti's understanding that empirical price-paths would generally be quite a bit simpler than those portrayed in the theoretical literature; and the fact that Marx's transformation problem as an iterative approximation procedure that can leads to a highly accurate nonlinear approximation of empirical Sraffian prices.

2. Toy models and actual empirical patterns

A stated purpose of Sraffa's book (1960, p. vi) was its aim to provide a basis for a critique of neoclassical theory. Early writers in the Sraffian tradition often produced small numerical examples, toy models, in which changes in the wage share (equivalently in the profit rate) gave rise to reswitching, reverse capital deepening and pronounced price curvature. These models were not informed by actual empirical patterns, the objective being to produce disturbing results within the bounds of neoclassical theory itself. Hence competition was essentially the same as perfect competition, equilibrium was assumed throughout, and all choices made by firms were assumed to be under equilibrium condition. But over time a growing body of empirical evidence found that actual relative prices of production were seldom highly curved and that wage-profit curves were mostly near-linear. Under such conditions, while the classical conflictual relation between wages and profit rates might generate an aggregate *pseudo* production function (to use Samuelson's own term) with price-generated pseudo marginal products approximately equal to corresponding "factor prices", the causation was from the latter to the former (Shaikh 2016, Ch. 9).

3. The Sraffa stochastic effect (SSE)

Sraffa's archives were recently made publicly available, and some of the material must have come as a shock to those Sraffians whose work had been dismissive of Marx (Steedman 1977). It became clear that Marx's writings had an important role in the project that culminated in Sraffa's *Production of Commodities by Means of Commodities* (1960), and that Sraffa continued to support key aspects of Marx's formulations. One essential point was that in capitalist production labour-power was itself a commodity, so that commodities are produced by "by labour out of commodities (D1/91/16; emphasis added)": hence production of commodities by means of commodities (Bellofiore and Carter 2014, 214). Further, there is the connection between Marx's and Sraffa's "standard" industries. In his treatment of the prices of production as transforms of labor values, Marx notes that that since individual production prices differ from labor values, these differences must be fed back onto individual costs of production which would then also deviate from their labor value equivalents. This is transformation via successive approximations. Marx goes on to argue that there would be one particular industry that would be exempt from price-value deviations because it had a special structure with a particular composition, an "ideal average, i.e., an average that does not really exist, i.e., a[n] *ideal as a standard* …In [which] … the price of production is ... the same or almost the same as the value, and the profit the same as the surplus value produced by" it (Marx 1967, Ch X, pp. 173-174, emphasis added). Sraffa's own Standard Industry is exactly such a composite industry, an ideal average that does not exist as such, whose price would remain equal to its labor value (its initial price) regardless of the level of the rate of profit (Sraffa 1960, pp. 12-23).

Sraffa also states that in light of the "statistical compensation of large numbers" (D3/12/35: 28) (Perri 2014, 109) it is obvious that the "propositions of M[arx] are based on the assumption that the composition of any large aggregate of commodities (wages, profits, constant capital) consists of a random selection, so that the ratio between their aggregates (rate of surplus value, rate of profit) is approximately the same whether measured at 'values' or at the prices of production corresponding to any rate of surplus value. . . . *This is obviously true*" (Bellofiore 2001, 369). In response to Eaton's (1960) review of the *Production of Commodities by Commodities*, Sraffa defends Marx again: "It is clear that M[arx]'s pro[position]s ... are based on the assumption (justified in general) that the aggregates are of some average composition. This is in general *justified in fact*, and since it is not intended to be applied to detailed minute differences it is all right ... This should be good enough till *the tiresome objector arises*. If then one must define which is the average to which the comp[osition] should conform for the result to be exact and not only approximate, it is the S[tandard] Comm[odity]." (Bellofiore 2014, pp. 220-221, emphasis added).

It turns out that Sraffa's Stochastic Effect (SE) is extremely powerful at the aggregate level. Using actual US input-output tables[1] for 1947, 1958, 1963, 1967, 1972 (71-order), my student Eduardo Ochoa's pathbreaking work (Ochoa 1984; Ochoa 1989) demonstrated that aggregate profit rates measured in

[1] US data has several advantages. The data is good, the time span is long, US economy is relatively closed (from 1977-2007 almost 90 percent of goods are produced domestically), and there are good domestic approximations to the input-output structure of imported commodities.

labor values and prices of production, respectively, are very close and that wage-profit curves tend to be generally smooth and near-linear – so that reswitching is likely to be rare. This is despite the fact that the composition of the actual output vector is very different from that of Sraffa's standard commodity (Ochoa 1984, Chs. V-VII). In my recent work, using Ochoa's tables and an additional 1998 (65-order) table, I calculated physical vectors of intermediate input, workers consumption, surplus product, net output and capital stock. The corresponding aggregates were circulating constant capital, variable capital, surplus value, value added, the rate of surplus value, the rate of profit, and the output-capital ratio, each measured at prices proportional to labor values (direct prices) and at prices of production corresponding to the observed rate of profit ($\mathrm{r_{obs}}$) in that year. Calculations were made for both circulating and fixed capital models[2].

The ratios of the aggregate production price of each bundle to its direct price are then calculated. Since Sraffa prices equal labor values at a zero rate of profit (r = 0), the preceding ratios also represent the effect on each variable of moving from r = 0 to the observed rate of profit $\mathrm{r_{obs}}$. In the circulating capital case, average price-value ratios for various aggregates range between 0.95 and 1.03 (Table 1), while in the fixed capital case they range between 0.97 and 1.05 (Table 2). As Marx and Sraffa had both understood, there is no effective difference between actual aggregates (Shaikh 2020)[3].

Insert here: Table 1: Ratios of Price and Value Aggregates at Observed Rates of Profit, Circulating Capital

Insert here: Table 2: Ratios of Price and Value Aggregates at Observed Rates of Profit, Fixed Capital

These considerations bring us back to the difference between a small arbitrary numerical (toy) model of the economy and a small representative model. I would argue that the smallest representative model should have three functionally distinct commodities: one material input (basic good) that enters into all sectors, one consumption good, and one capital good (machine). Then the input–output matrix would have two zero rows corresponding to the consumption and capital goods, since neither of these enter into production as inputs; and the capital goods matrix K would also have two zero rows, because neither the material input or the consumption good function as capital goods. It is easy to show that such a system will have linear Sraffian prices because the subdominant eigenvalues of the vertically integrated capital stock matrix $K(I-A)^{-1}$ will be zero (Shaikh 2016, p. 442). This is in fact the structure of the Schemes of Reproduction worked out by Marx in the 1870s and published by Engels in Volume II of *Capital* in 1884 (Marx 1967, Preface by Engels p. 4, Chs. 20-21).

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[2] Sources and methods for the 1947-1972 data are in Shaikh (1998) Appendix 15.2, and for 1998 are in Shaikh (2012) Data Appendix. In order to isolate the effects of changes in *relative* prices on various aggregate bundles, we must keep the price *level* constant. Total market price is defined as the market value of gross output, and total direct price and total price of production at any given profit rate are scaled to match total market price. This keeps the value of money the same across all three types of prices.

[3] The aggregate correspondences also hold for valuations at market prices (Shaikh 2016, Ch. 9, Sections V-IX).

4. Empirical properties of Sraffa prices

On the issue of price-value differences, Pasinetti has said that "the *directions* of the divergences of prices from 'values' cannot be predicted with absolute certainty on the basis of the 'organic composition' alone". Nonetheless, "in the neighborhood of any given profit rate a probable but not absolutely certain indicator is provided by the degree of capital intensity of the various processes of production". Part of the problem, he notes, is that the capital-intensity is itself based on the aggregate price of an industry's bundle of capital goods, so that it too varies with the profit rate (Pasinetti 1977, pp. 24, 83-84, footnote 16 p. 83, 136). We may pose the problem in the following manner: at a profit rate near zero, prices are basically equal to labor values, so an industry's price-value ratio is equal to 1 and its capital intensity is the same in price and value terms. If this initial direction is to be maintained as the profit rate varies, the industry's price-value ratio may change direction and may even fluctuate in direction (wiggle) but must not cross below its starting point of 1. Linear or near-linear price paths will have this property, while curvilinear paths may or may not.

In Shaikh (1998, pp. 236-242) I found that in five 71-order input-output tables of 1947, 1958, 1963, 1967, and 1972, encompassing 355 prices in total, the price-value ratio curves are "almost invariably linear". Since Sraffa prices equal labor values at r = 0, all curves begin at 1. As the rate of profit increases from zero to the maximum rate, 349 curves (98 percent) remain on their initial side – i.e. maintain their initial direction of deviation. Only 6 switch directions, and in each case the price-value deviations are very small throughout, of the order of 2-3 percent. In such cases, because the industry's organic composition of capital is very close to that of the standard industry, the price effects generated by a changing rate of profit overwhelms the initial capital-intensity effect. in Shaikh (2012) I showed mathematically that if individual industry output-capital curves are linear, the corresponding price curves will also be linear. I found that in the 65-order 1998 table, 61 (94 percent) of the individual industry output-capital ratios are very close to linear, which is why the corresponding price curves are near-linear. Only 4 sectors have output-capital ratios that switch direction, and this only in a very small degree at rates of profit near the maximum rate of profit. Such findings strongly validate Pasinetti's sense of the mechanics of price curves.

In the present paper, I extend this investigation to a much larger matrix, the 403-order 2002 US input-output (IO) table[4]. An underlying question is whether a much larger matrix will exhibit very different patterns. The answer is that going from a 65-order to 403-order does not make much of a difference. 329 (82 percent) of the price-curves are near-linear of the forms depicted in Figure 1[5]. We can see that near-linear curves can range quite far from their (common) starting point. Figure 2 displays an illustrative sample of the 24 (6 percent) non-linear curves, and here we see that the range of variation is much smaller. This comes out sharply when we put the samples from Figures 1 and 2 on the same chart, as in Figure 3. Finally, Figure 4 shows the only 4 (1 percent) curves qualify as "wiggly"[6]. As with the non-linear curves, the range of variation is very small. It should be noted that when the variation is small, as

[4] The construction of the 2002 input-output matrix is detailed in Shaikh, Coronado et al. (2020, Appendix I). I thank Luis Daniel Torres Gonzalez for his great help on this data, and Amr Ragab for his help with data visualization.

[5] Some individual curves may fall on the cusp of categories such as near-linear, non-linear, etc., but I would argue that the illustrative shapes shown are distinct.

[6] The small-scale directional changes in these curves seem to be an artifact of the 0.05 step size used for r/R. Even with this size, there are almost 8,500 observations.

it is in these curves, one observes minor fluctuations all along the curve. These appear to be artifacts of the calculations, since even plots of theoretical straight lines exhibit such ripples (see Figures 2, 4, 7).

Insert here: Figure 1: Illustrative near-linear price-value curves, US 2002 403-order IO table

Insert here: Figure 2: Illustrative non-linear price-value curves, US 2002 403-order IO table

Insert here: Figure 3: Illustrative near-linear and non-linear price-curves, US 2002 403-order IO table

Insert here: Figure 4: Illustrative "wiggly" price-value curves, US 2002 403-order IO table

5. An empirically sufficient form for Sraffa prices

In section 3. I noted that Marx's transformation procedure is actually an iterative one that can be shown to converge on Sraffa prices (Shaikh 1977). Assuming only circulating capital for simplicity, and denoting vectors and matrices in bold, let $\boldsymbol{I}, \boldsymbol{A}, \boldsymbol{l}$ represent the identity matrix, the input-output matrix and the labor coefficients vector, respectively. Let $\boldsymbol{v}, \boldsymbol{b}$ represent the labor value vector and the workers' consumption goods vector. Then unit labor value is the sum of the labor value of inputs $\boldsymbol{vA}$ and the living labor $\boldsymbol{l}$ added by workers, and the unit surplus value vector $\boldsymbol{s}$ is the excess of living labor $\boldsymbol{l}$ added and the labor value of workers' consumption $\boldsymbol{vb}$.

1) $\boldsymbol{v} = \boldsymbol{vA} + \boldsymbol{l} = \boldsymbol{vA} + \boldsymbol{vb} + \boldsymbol{s}$
2) $\boldsymbol{v} = \boldsymbol{l}(\boldsymbol{I} - \boldsymbol{A})^{-1}$ (from equation 1)

The first step in Marx's transformation procedure is to create a new price vector $\boldsymbol{p}^{(1)}$ in which unit surplus value $\boldsymbol{s}$ is replaced by unit profit $\boldsymbol{\pi}^{(1)}$ reflecting a common profit rate $r^{(1)}$. Let $\boldsymbol{X}$ equal the total output vector. Then in the first round $r^{(\boldsymbol{1})} = \frac{\boldsymbol{sX}}{(\boldsymbol{vA}+\boldsymbol{vB})\boldsymbol{X}}$ and the new price vector is

3) $\boldsymbol{p}^{(1)} = \boldsymbol{vA} + \boldsymbol{vb} + r^{(1)}(\boldsymbol{vA} + \boldsymbol{vB})$

Marx then goes on to say that the next step (which he does not pursue) would be to feed back the new prices onto costs of production. Keeping output prices unchanged for the moment, this would yield new costs and define new unit profits vector $\boldsymbol{\pi}^{(2)}$ that can in turn be used to define a common average rate of profit $r^{(2)}$ and so on. Shaikh (1977) proves that this process converges to Sraffa prices and uniform rate of profit r.

4) $\boldsymbol{p}^{(1)} = \boldsymbol{p}^{(1)}\boldsymbol{A} + \boldsymbol{p}^{(1)}\boldsymbol{b} + \boldsymbol{\pi}^{(2)}$

5) $\boldsymbol{p}^{(2)} = \boldsymbol{p}^{(1)}\boldsymbol{A} + \boldsymbol{p}^{(1)}\boldsymbol{b} + r^{(2)}(\boldsymbol{p}^{(1)}\boldsymbol{A} + \boldsymbol{p}^{(1)}\boldsymbol{b})$

Marx's first step in equation 3 constitutes a linear approximation. Sraffa develops the complete system of prices of production in which the wage share $w = 1 - \frac{r}{R}$ is expressed in terms of the standard commodity, r is the (endogenous) profit rate and R is the maximum rate of profit. Assuming that wages are not advanced so that only the materials costs appears as circulating capital, the price vector $\boldsymbol{p}$ can be written as total cost $(\boldsymbol{wl} + \boldsymbol{pA})$ plus profit on material costs: $\boldsymbol{p}(r) = \boldsymbol{wl} + \boldsymbol{p}(r)\boldsymbol{A} + r\boldsymbol{p}(r)\boldsymbol{A} = \left(1 - \frac{r}{R}\right)\boldsymbol{l} + \boldsymbol{p}(r)\boldsymbol{A} + r\boldsymbol{p}(r)\boldsymbol{A}$. From equation 2, $\boldsymbol{v} = \boldsymbol{l}(\boldsymbol{I} - \boldsymbol{A})^{-1}$, and defining $\boldsymbol{H} = \boldsymbol{A}(\boldsymbol{I} - \boldsymbol{A})^{-1}$ we get a condensed form of Sraffa prices (equation 6) that we can rewrite in an expanded form (equation 7)

6) $\boldsymbol{p}(r) = \left(1 - \frac{r}{R}\right)\boldsymbol{v} + r\boldsymbol{p}(r)\boldsymbol{H}$

7) $\boldsymbol{p}(r) = [\boldsymbol{v} + r\boldsymbol{v}\left(\boldsymbol{H} - \frac{1}{R}\boldsymbol{I}\right)] + r(\boldsymbol{p}(r) - \boldsymbol{v})\boldsymbol{H}$

Sraffa prices in equation 6 start at labor values $\boldsymbol{v}$ when the profit rate $r = 0$ and at $r = R$ they end at the price vector defined by $R\boldsymbol{p}(R)\boldsymbol{H} = \boldsymbol{p}(R)$, where $\boldsymbol{p}(R)$ is the dominant left-hand eigenvector of $\boldsymbol{H}$ and $\frac{1}{R}$ is the corresponding dominant eigenvalue, both structurally defined by $\boldsymbol{H}$.

Shaikh (1998, p. 237) treats the term in square brackets in equation 7 as a Marxian linear approximation defined in terms of vertically integrated organic compositions of capital and finds that it can account for 98 percent of variation in Sraffa prices. However, while this linear approximation starts from labor values v when $r = 0$ (as do both Marx's own first approximation and Sraffa prices) it ends at the price vector $R\boldsymbol{vH}$ which is different from the end point $\boldsymbol{p}(R)$ of Sraffa prices.

The second term in the Sraffa price equation 6 is $r\boldsymbol{p}(r)\boldsymbol{H}$, in which both r and $p(r)$ vary along the price path: this what gives rise to the complexity of individual price movements (Pasinetti 1977, pp. 82-84). Bienenfeld (1988) proposes to replace this second term with $r\boldsymbol{p}(R)\boldsymbol{H}$. Since $\boldsymbol{p}(R)$ is fixed, equation 8 is a linear approximation that starts and ends at the same point as actual price curves.

8) $\boldsymbol{p}(r)^{(1)} = \left(1 - \frac{r}{R}\right)\boldsymbol{v} + r\boldsymbol{p}(R)\boldsymbol{H}$

From equation 8, $\left(1 - \frac{r}{R}\right)\boldsymbol{v}$ = $\boldsymbol{p}(\boldsymbol{r})^{(\boldsymbol{1})} - r\boldsymbol{p}(R)\boldsymbol{H}$ so we can write the overall Sraffa price equation 6 as

9) $\boldsymbol{p}(r) = \boldsymbol{p}(r)^{(1)} + r(\boldsymbol{p}(r) - \boldsymbol{p}(R)\boldsymbol{H}$

In the spirit of Marx's iteration procedure, we may now construct a second iterate by feeding the linear price approximation back onto vertically integrated input costs to arrive at equation 10. As we can see, the second iterate, *a quadratic approximation*, starts at $\boldsymbol{p}(0)^{(1)} = \boldsymbol{v}$ at $r = 0$ and ends at $\boldsymbol{p}(R)$ at $r = R$. It also has the same initial slope as the Sraffa price curve (Bienenfeld 1988, p. 251). Hence both approximations start and end at the same points as Sraffa prices. It must be stressed that *these linear and quadratic forms are entirely structural,* not statistical fits.

10) $\boldsymbol{p}(r)^{(2)} = \boldsymbol{p}(r)^{(1)} + r(\boldsymbol{p}(r)^{(1)} - \boldsymbol{p}(R))\boldsymbol{H}$

We are now ready to compare the first and second structural approximations to the various types of empirical price curves previously considered in Figures 1-4. Figure 5 shows that in the case of two representative near-linear curves, the linear approximations are very good and the quadratic ones are excellent. Figure 6 displays a representative non-linear curve and its linear approximations and quadratic approximations. The quadratic is clearly better, but first glance the fits do not appear as good as they did against near-linear curves. But this is a scale-effect due to the small range of the curve. The *maximum* difference between the curve and its quadratic approximation is only 2 percent and only 8 percent in the case of the linear approximation. Putting non-linear and near-linear curves on the same scale as was done in Figure 3 dispels the illusion of substantial differences. All told, the quadratic approximation is an excellent fit *for 99 percent of the 403 price curves*. Similar results have been found across five European countries, but on the much smaller scale of 51-58 order tables (Iliadi, Mariolis et al. 2014).

Insert here: Figure 5: Near-linear price-value curves and approximations, US 2002 403-order IO table

Insert here: Figure 6: Non-linear price-value curve and approximations, US 2002 403-order IO table

This leaves the approximations of the 4 wiggly curves, a mere 1 percent of the total, as illustrated in Figure 7. Given the direction-reversal of these price curves, a quadratic approximation cannot fully mimic them. Even so, the maximum error is on the order of 2 percent. It should be mentioned that when curves span some distance, as in Figures 1, 3, 5, their plots appear smooth. As previously noted, when the span is small as in Figures 2, 4, 7, there are very small fluctuations all along the curve. This is an artifact of the calculations, since even the plots of the linear approximation in Figures 6,7 that should appear as exact lines exhibit these small fluctuations.

Insert here: Figure 7: Wiggly price-value curve and approximations, US 2002 403-order IO table

Given the Stochastic Effect and the shapes of individual price curves, it is to be expected that the aggregate wage-profit curve would be near-linear, as has been found in so many other studies reported in Shaikh (2016, Ch. 9). On this same reasoning, we would also expect the output-capital ratio to move smoothly between its bounds. Figure 8 depicts the wage-profit curve, and Figure 9 the output-capital ratio, both of which are indistinguishable from their quadratic counterpart. It follows that the quadratic approximation is a powerful and empirically sufficient structural form of price, wages and output-capital ratios.

Insert here: Figure 8: Wage-profit curve and approximations, US 2002 403-order IO table

Insert here: Figure 9: Output-capital ratio and approximations, US 2002 403-order IO table

6. Summary and Conclusions

This paper considers three linked propositions. First, in his archival notes Sraffa insists that Marx was "of course" right in thinking that the "statistical compensation of large numbers" would make price and labor value aggregates essentially the same at the empirical level. This is exactly what we find in five different US input-output tables. Aggregates of circulating constant capital, variable capital, surplus value, value added, the rate of surplus value, the rate of profit, and the output-capital ratio, measured at prices proportional to labor values (direct prices) and at prices of production defined by the observed rate of profit, range between 0.95 and 1.03 in a circulating capital model and between 0.97 and 1.05 in a fixed capital model. Marx and Sraffa are completely correct. By implication, representative small models that pretend to be empirically relevant should reflect such properties.

Second, Pasinetti's is correct to caution us that actual price-paths are quite a bit simpler than those portrayed in the theoretical literature. This is fully confirmed by the near-linear shapes of the 403-industry price-curves generated from 2002 matrix.

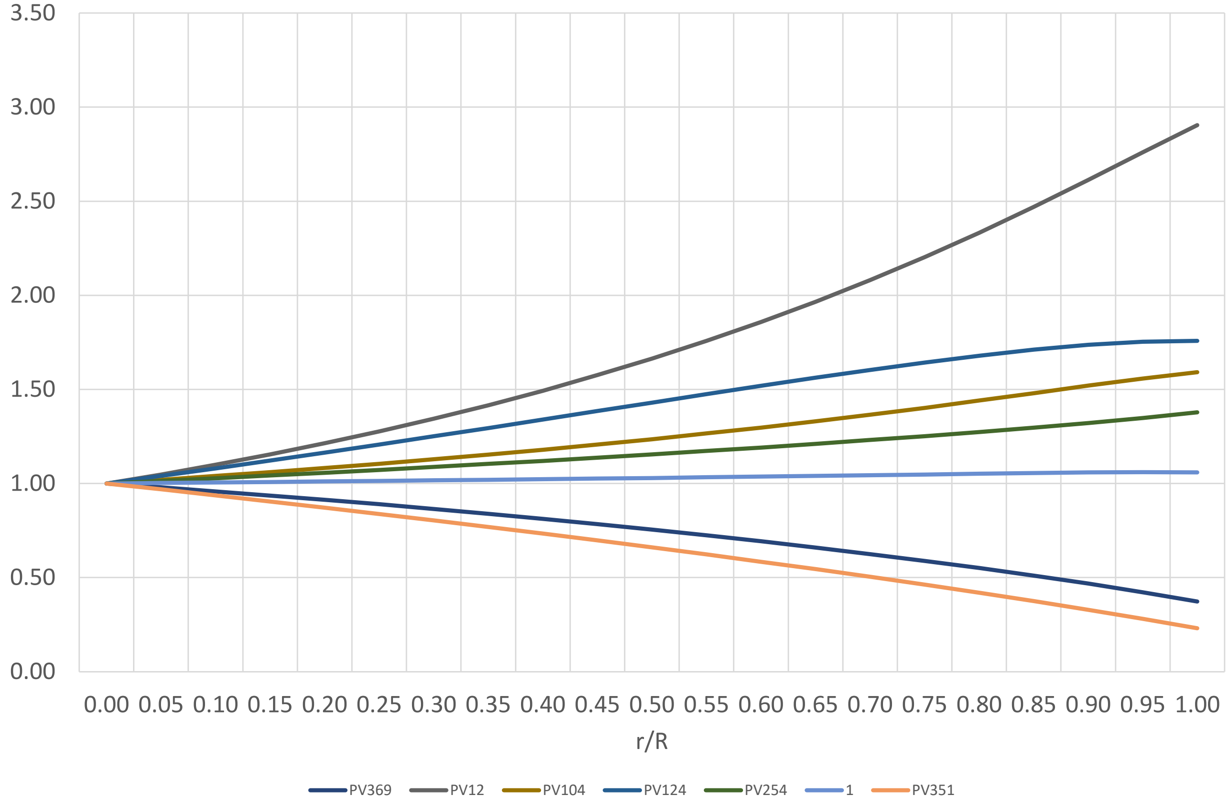
Third, Marx presents his transformation procedure as an iterative process. It has been shown that this procedure leads to successively better approximations that converge on full Sraffa prices. Bienenfeld constructs a linear first approximation and a non-linear quadratic second approximation. Both forms are entirely structural, not statistical fits. Each approximation begins and ends at the same points as do the corresponding individual price curve. The linear form turns out to be a good approximation of the 403 curves, but the quadratic is truly excellent. I would therefore argue, on geometric and empirical grounds, that Bienenfeld's simple nonlinear expression is an empirically sufficient general form of Sraffa prices.

Data Tables and Figures

Table 1: Ratios of Price and Value Aggregates at Observed Rates of Profit, Circulating Capital							
Matrix Size	71	71	71	71	71	65	
	1947	**1958**	**1963**	**1967**	**1972**	**1998**	***Average***
Observed rate of profit (r_{obs})	*0.32*	*0.32*	*0.36*	*0.40*	*0.36*	*0.23*	*0.33*
Observed profit share $(\frac{r_{obs}}{R})$	*0.32*	*0.30*	*0.35*	*0.39*	*0.35*	*0.22*	*0.32*
Constant Capital	1.02	1.02	1.03	1.04	1.03	1.02	*1.03*
Variable Capital	0.99	0.98	0.97	0.97	0.96	0.99	*0.98*
Surplus Value	0.98	0.98	0.97	0.97	0.98	0.98	*0.98*
Value Added	1.00	0.98	0.97	0.97	0.97	0.98	*0.98*
Rate of Surplus Value	1.03	1.00	1.00	1.00	1.02	0.99	*1.01*
Rate of Profit	0.97	0.96	0.94	0.94	0.95	0.96	*0.95*
Maximum Rate of Profit (R)	0.97	0.96	0.94	0.94	0.94	0.96	*0.95*

Table 2: Ratios of Price and Value Aggregates at Observed Rates of Profit, Fixed Capital							
Matrix Size	71	71	71	71	71	65	
	1947	**1958**	**1963**	**1967**	**1972**	**1998**	***Average***
Observed rate of profit (r_{obs})	*0.24*	*0.18*	*0.21*	*0.23*	*0.19*	*0.11*	*0.19*
Observed profit share $(\frac{r_{obs}}{R})$	*0.29*	*0.26*	*0.28*	*0.31*	0.28	*0.27*	*0.28*
Constant Capital	1.02	1.03	1.04	1.04	1.03	1.03	*1.03*
Variable Capital	0.98	0.99	0.98	0.99	0.98	1.01	*0.99*
Surplus Value	0.98	0.96	0.95	0.94	0.96	0.98	*0.96*
Value Added	0.98	0.97	0.97	0.97	0.97	0.96	*0.97*
Rate of Surplus Value	0.99	0.97	0.97	0.96	0.97	1.00	*0.98*
Rate of Profit	1.04	1.04	1.05	1.05	1.04	0.98	*1.04*
Maximum Rate of Profit (R)	1.04	1.06	1.07	1.08	1.06	1.00	*1.05*

Figure 1: Illustrative near-linear price-value curves, US 2002 403-order IO table



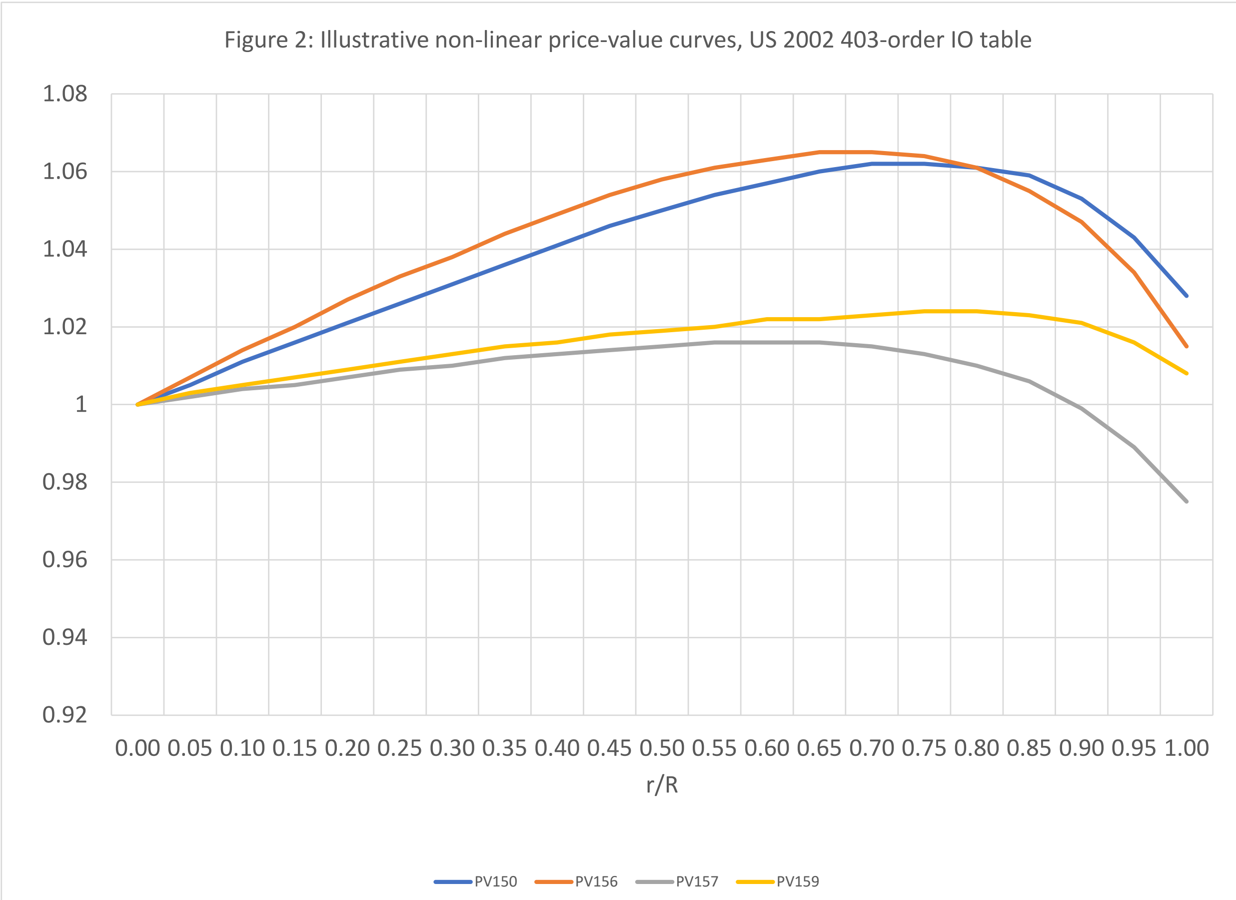
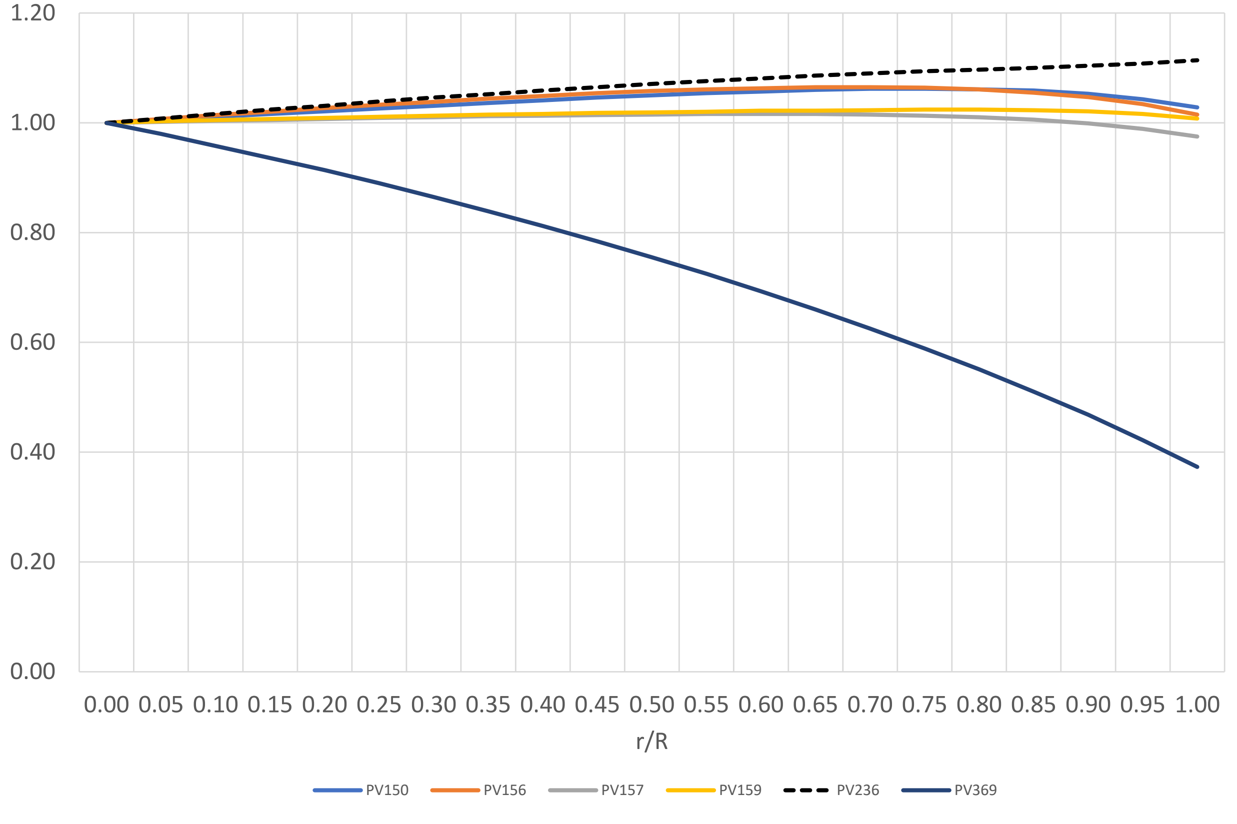


Figure 2: Illustrative non-linear price-value curves, US 2002 403-order IO table
1.08
1.06
1.04
1.02
1
0.98
0.96
0.94
0.92
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
r/R
PV150
PV156
PV157
PV159

Figure 3: Illustrative near-linear and non-linear price-curves, US 2002 403-order IO table



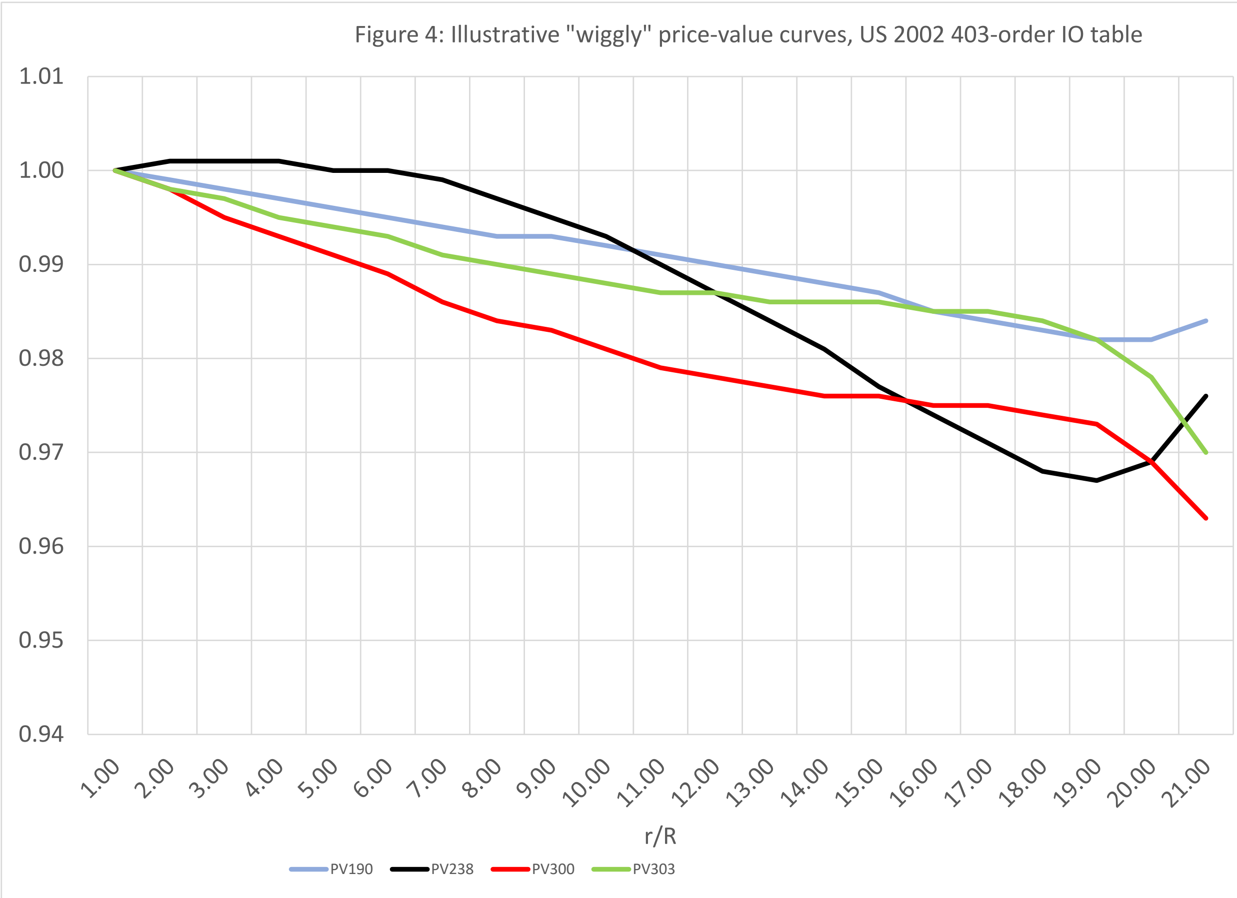
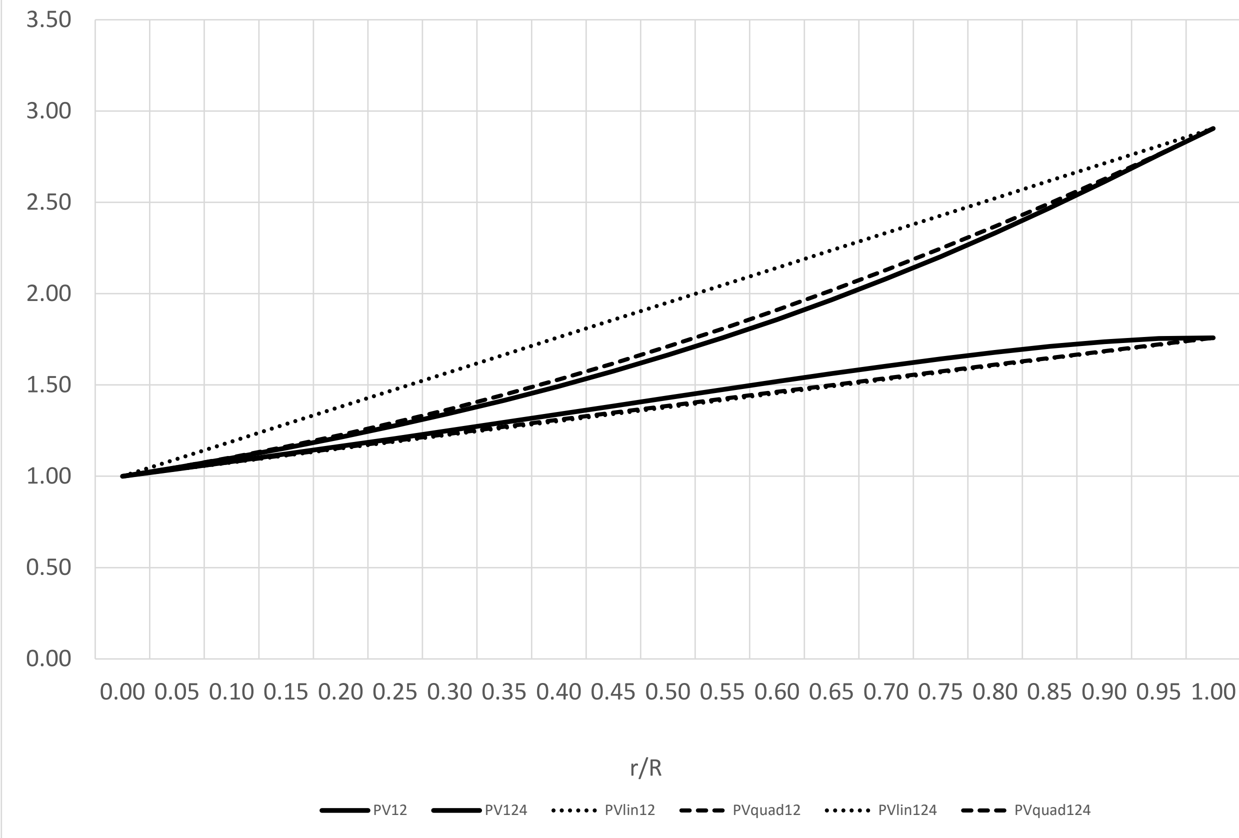


Figure 4: Illustrative "wiggly" price-value curves, US 2002 403-order IO table
1.01
1.00
0.99
0.98
0.97
0.96
0.95
0.94
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
9.00
10.00
11.00
12.00
13.00
14.00
15.00
16.00
17.00
18.00
19.00
20.00
21.00
r/R
PV190
PV238
PV300
PV303

Figure 5: Near-linear price-value curves and approximations, US 2002 403-order IO table



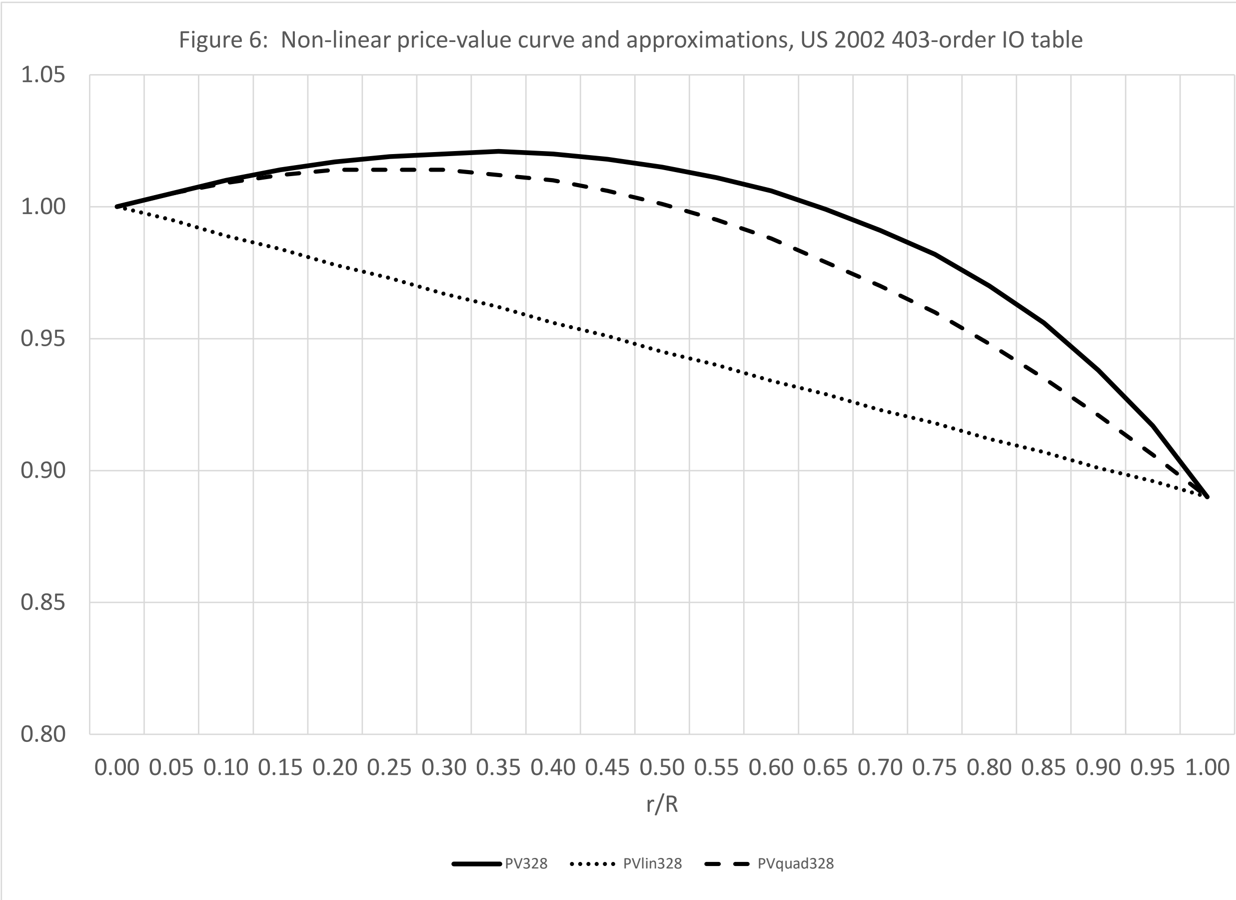


Figure 6: Non-linear price-value curve and approximations, US 2002 403-order IO table
1.05
1.00
0.95
0.90
0.85
0.80
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
r/R
PV328
PVlin328
PVquad328

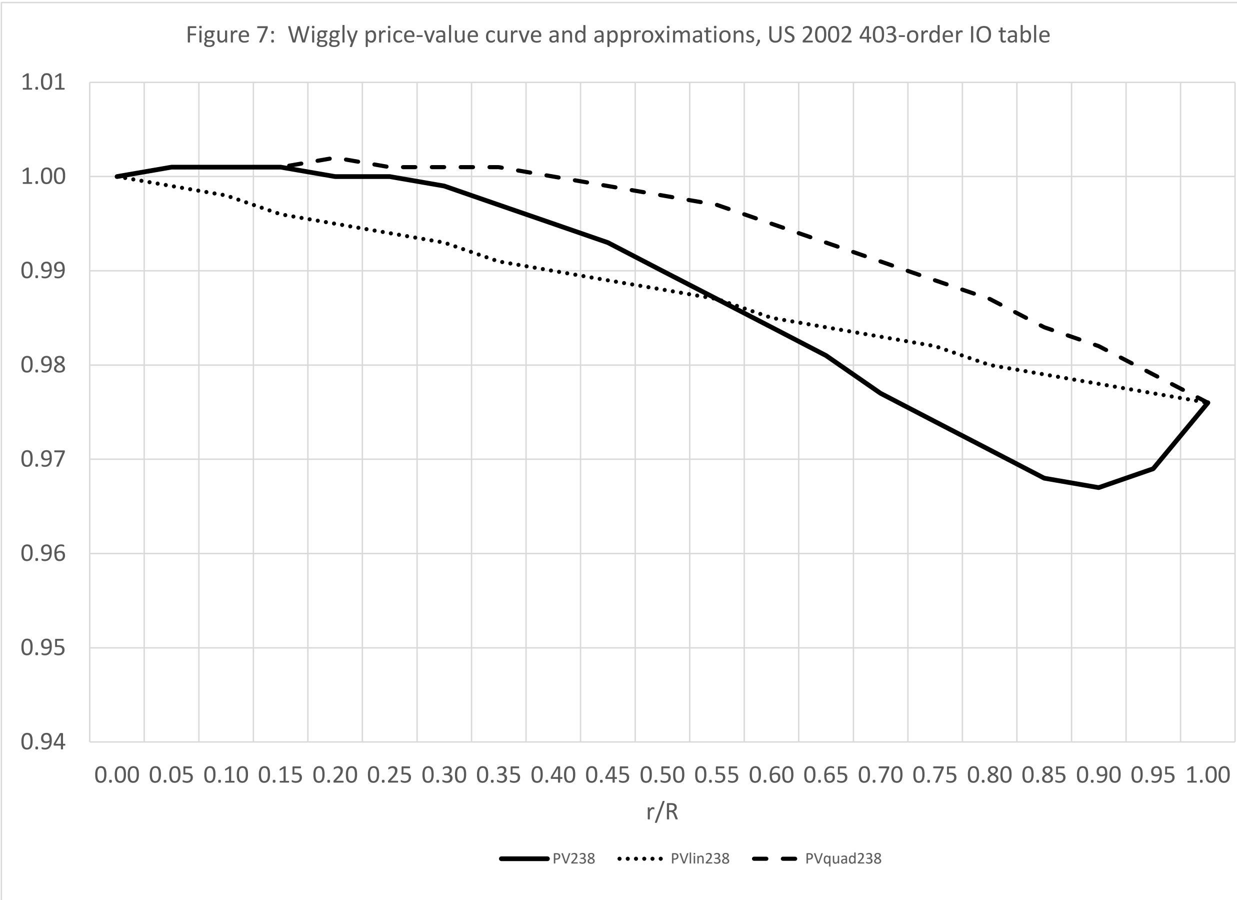


Figure 7: Wiggly price-value curve and approximations, US 2002 403-order IO table
1.01
1.00
0.99
0.98
0.97
0.96
0.95
0.94
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
r/R
PV238
PVlin238
PVquad238

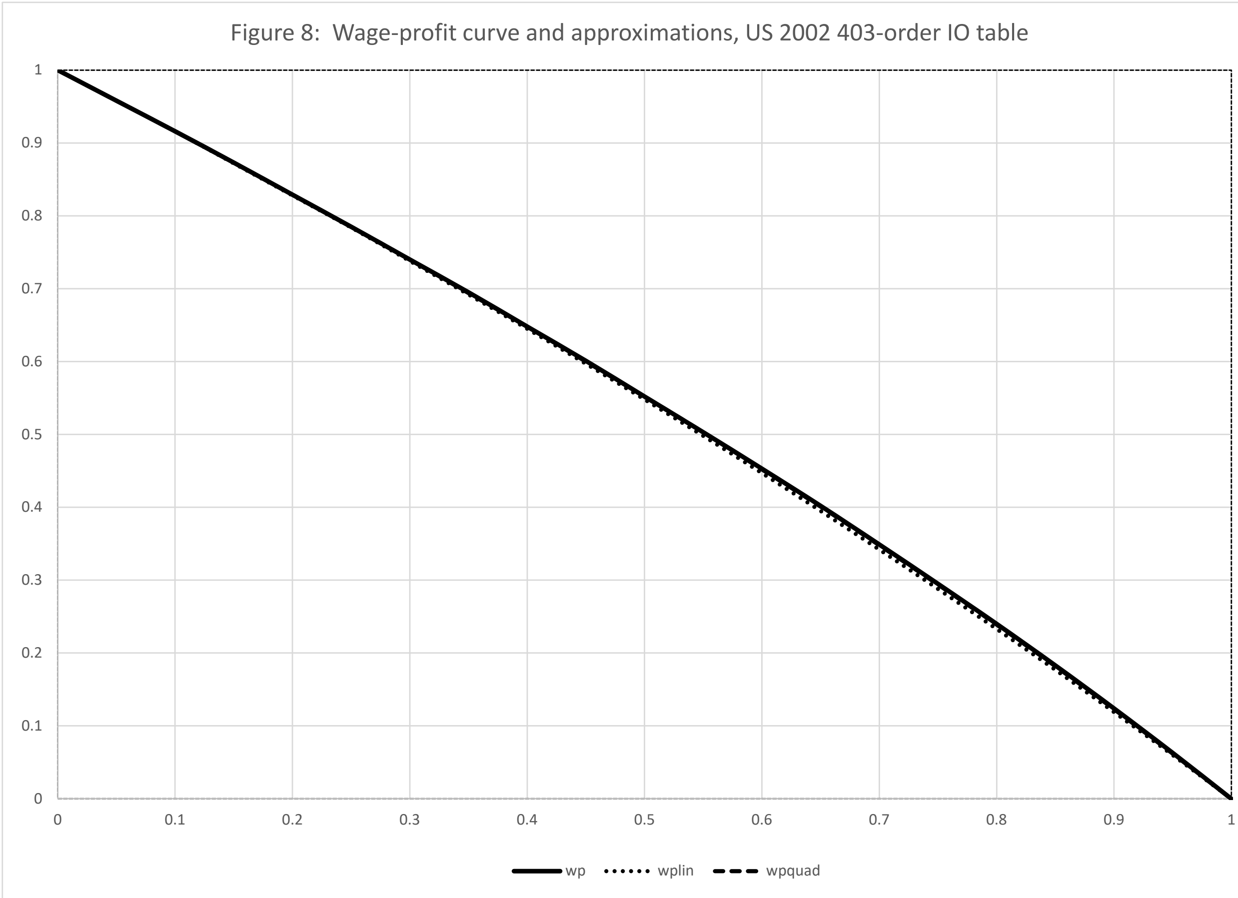


Figure 8: Wage-profit curve and approximations, US 2002 403-order IO table
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
wp
wplin
wpquad

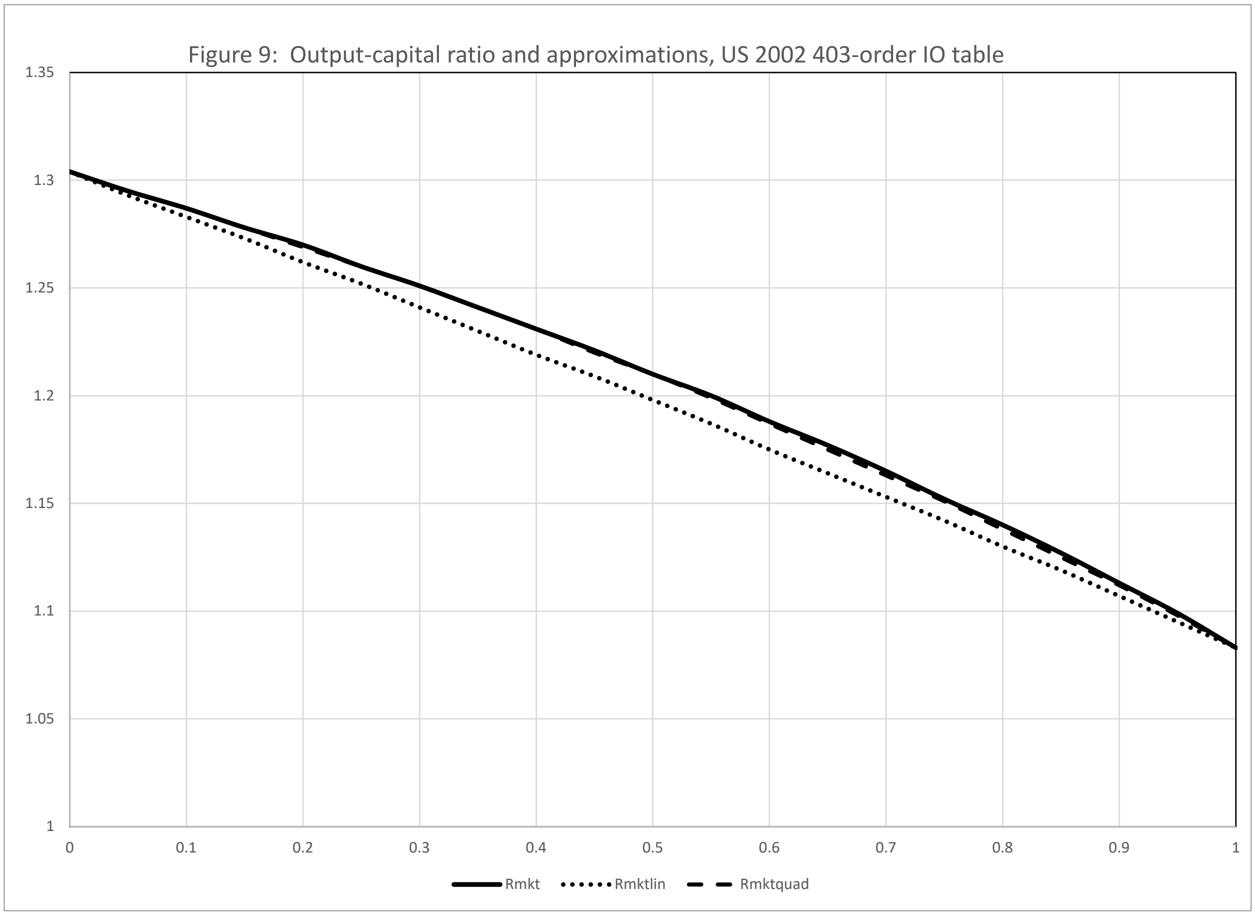


Figure 9: Output-capital ratio and approximations, US 2002 403-order IO table
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Rmkt
Rmktlin
Rmktquad

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