18 Wandering Around the Warranted Path: Dynamic Nonlinear Sdlutions to the Harrodian Knife-Edge*

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1 INTRODUCTION

Classical economics conceived of capitalism as an inherently expansive system which was ultimately regulated by its level of profitability. This approach reached its highest development in the works of Marx and Schumpeter, with their portrayal of a system driven by its inner mechanisms along erratic and periodically unstable paths of accumulation (Bleaney, 1976, ch. 6; Garegnani, 1978, pp. 183-5; Shaikh, 1984, section II). In what follows, I will refer to this overall perspective as the classical tradition.

Dynamic analysis of the above sort is typically constructed in terms of various sets of gravitational processes operating at intrinsic speeds ranging from the fairly fast to the very slow. For instance, a discrepancy between aggregate demand and supply produces a faster response than that between aggregate supply (output) and capacity (potential output), because the inventory and production level adjustments associated with the former are fairly rapid in comparison to the fixed capital and capacity level adjustments associated with the latter. This is why aggregate demand/supply adjustments arc generally treated as as 'short-run' while the aggregate supply/capacity adjustments are 'long-run" (Kaldor, 1960, pp. 31-3). But the notion of fast and slow adjustments is broader than the conventional macroeconomic distinction between short run and long run, for two reasons.

First of all, the fact that there are fast and slow adjustment processes does *not* imply that these processes lead to corresponding states of equilibrium. For instance, if a discrepancy between aggregate demand and supply generates a response in aggregate production, demand and prices, which in

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turn feed back to modify the initial discrepancy, and so on, in such a way that aggregate supply and demand end up gravitating around a mutual state of balance, this need not imply that demand and supply will end up equal. It is sufficient to imagine that the demand and supply fluctuate endlessly *around* their balance point without ever coming to rest on it. Supply would then approximately equal demand over some average period of oscillation. Yet at any moment of time, each would differ from the corresponding balanced amount. A similar argument could be made for the slow adjustment between supply and capacity. These are the kind of gravitational processes which are implicit in Marx's conception of a balance point as a 'regulating average', as opposed to some attained-and-held 'equilibrium state.[2]

Secondly, the fact that the fast and slow adjustment processes gravitate (orbit) around some regulating averages docs not imply, as it does in Kaleckian and Keynesian constructions, that the fast adjustment defines some average level of output and employment, so that growth only enters the picture in the slow adjustment process. On the contrary, *growth is a part of the environment of both processes*.[3] The fast adjustment process defines time paths for aggregate demand and supply, not levels, and the slow adjustment process modifies (modulates) these paths in the light of their average results.

Since the fast adjustment process operates within the context of accumulation, the average levels of demand, supply and capacity change over time. The supply and capacity paths therefore define a corresponding path of capacity utilization (the ratio of actual output to capacity). But there is no presumption within the Classical tradition that the fast process will cause actual output to gravitate around capacity. If we think of the fast process as operating in time units called (say) weeks, and the slow one in time units called (say) years, then the fast process will produce an annual level of capacity utilization u which will generally differ from the normal' rate u_n (= 1, by construction).[4]

The conclusion that the fast process roughly equalizes aggregate supply and demand, but not aggregate output and capacity, leads automatically to the consideration of the effects of any resulting discrepancy between actual and normal levels of capacity utilization. This is precisely the focus of the slow adjustment process in the Classical tradition, in which the above discrepancy is assumed to react back upon the rate of accumulation, thus altering the paths of actual output and capacity, modifying the initial discrepancy, which feeds back onto accumulation, and so on, all at the relatively slow pace consistent with the longer time horizon inherent in this process.

The Classical tradition implicitly assumed that this slow gravitational process was stable, in the specific sense that it led to the fluctuation of actual capacity utilization around some normal level. With this assumption, the basic groundwork was laid. Aggregate supply was thought to fluctuate around aggregate demand over some relatively fast process, and the resulting average aggregate output around the corresponding aggregate capacity over some relatively slow one. Classical dynamics was then able to concentrate on the properties of the normal capacity utilization path itself, and on the 'magnificent dynamics' arising from the still slower feedbacks between technical change, population growth and long run trends (Baumol, 1959, part I).

We have already noted that some distinction between fast and slow processes is common to all major traditions in economics, most often in the form of a distinction between short run and long run equilibrium states (as opposed to gravitational processes). More interestingly, *all major traditions implicitly or explicitly share the Classical notion thnt aggregate supply and demand are roughly balanced over some relatively fast adjustment process. Neo-Classical* economics not only assumes that aggregate supply and demand balance in short run equilibrium, but also that this balance point simultaneously corresponds to the short run "full employment" of available industrial capacity and labor power, which in this context means the absence of any involuntary excess capacity or unemployment. Keynesian and Kaleckian theories also typically assume that aggregate supply and demand balance in some short run equilibrium, but insist that this is perfectly consistent with involuntary excess capacity and labor unemployment (Kalecki, 1968, p. 182; Keynes, 1936, ch. 3).

Insofar as Keynesians insist the fast adjustment process balances aggregate demand and supply but not output and capacity or employment and labor force, their overall conclusions are actually very similar to those of the Classical tradition. It is in their respective characterizations of the slow adjustment process, in which any discrepancy between actual and normal capacity utilization feeds back onto the level of accumulation, that a great difference arises. The Classical tradition, as we have already noted, tended to assume that this slow adjustment process was stable. But Keynesians have no such luxury, for Harrod long ago derived 'the rather astonishing' result that the slow adjustment process is very unstable (Daumol, 1953, p. 44). In particular, any initial discrepancy between actual and normal rates of capacity utilization feeds back on accumulation in such a way as to exacerbate the problem: the normal capacity utilization (warranted) path is knife-edge unstable. In spite of many attempts to solve Harrod's problem, it persists to the present day.

In this paper, I will argue that the Harrod's apparently inescapable conclusion is, so to speak, quite unwarranted. The secret to the knife-edge lies in an unnoticed contradiction between his static specification of short run balance and his subsequent attempts at dynamics. As we shall see, once this error is corrected, it is easy to solve the knife-edge problem. The result is a Classical slow adjustment process in which the economy wanders

around the warranted path as the actual level of capacity utilization cycles around the normal level. In what follows, we will assume that aggregate demand and supply are roughly equalized over some fast process (as modeled in Shaikh, 1989, 1991), so that like Harrod we may concentrate on the slow process.

2 AGGREGATE DEMAND AND SUPPLY

In keeping with the Harrodian formulation, we will take money prices and wages to be constant, so that all quantities are effectively in real terms. But we begin our accounting with total output and total demand, as in Marxian and input-output accounts. rather than the more familiar net measures of national income accounts, because this will enable us to locate a crucial omission in the conventional definition of the latter.

Since we are ultimately concerned with dynamic analysis, it is important to take note of the fact that production takes time. Following the Classical traditional, we will define the unit of time as being equal to the average period of production. Inputs purchased in period $t-1$ then lead to output in period t. The money value of total aggregate supply Q_t at time t can then be written as the sum of materials costs M_{t-1}, labor costs W_{t-1}, and depreciation DEP_{t-1} on fixed capital, all stemming from inputs used in the actual production of this output, plus the potential profit on production P_t, which is by definition the residual (Robinson, 1966, p. 41; Godley and Cripps, 1983, p. 75, 1). This gives us the standard expression for output

$$Q_t = M_{t-1} + W_{t-1} + DEP_t + P_t \tag{1}$$

In period t, total aggregate demand D_t (sales) will be composed of current capital expenditures on materials M_t, on desired additions to finals goods inventories (desired inventory investment) IV_t, on new plant and equipment (gross fixed investment) IG_t, and of consumption expenditures on workers' and capitalists' consumption goods CW_t and CR_t, respectively.

$$D_t = M_t + IV_t + IG_t + CW_t + CR_t \tag{2}$$

Finally, we will define excess demand E_t as the difference between total demand D_t and total normal supply Q_t. Any discrepancy between supply and demand will then be reflected in undesired changes in final goods inventories $UCINV_t$. When excess demand is positive, the final goods inventories will be run down below their desired levels, so that the undesired change is negative. Thus

$$E_t = D_t - Q_t = \quad UCINV_t \tag{3}$$

Equations 1-3 allow us to derive the standard accounting identity that total output equals the sum of total sales and the undesired change inventories of final goods.

$$Q_t = M_t + Iv_t + IG_t + UCINV_t + (CW_t + CR_t) \tag{4}$$

The left hand side of equation 4 is total output, and the right hand side is its total distribution. Capital outlays for materials M_t, inventories Iv_t, and plant and equipment, IG_t represent gross additions to stocks of productive capital, $UCINV_t$ represents involuntary changes in the stock of final goods, and $CW_t + CR_t$ represent goods transferred to the personal stocks of consumers.

The next step is to derive net output. By definition, net output is the difference between total current output and that portion of capital outlays which represents the equivalent of materials and fixed capital used up in the previous period. But the use of materials in the previous period is M_{t-1}, since that is that amount used up as input into current production. If we designate the corresponding retirements (scrapping) of fixed capital by KR_{t-1}, then from equations 1-4 we can write net output as

$$Y_t = Q_t \quad (M_{t-1} + KR_{t-1})$$

$$Y_t = (Im_t + Iv_t + If_t + UCINV_t) + C_t \tag{5}$$

where Iv_t, and $UCINV_t$ are as defined previously, and

$Im_t = (M_t - M_{t-1})$ = net investment in materials
$Ik_t = (IG_t - KR_{t-1})$ = net investment in fixed capital
$C_t = CW_t + CR_t$ = total personal consumption

Equation 5 above is simply an accounting identity for net output. It does not assume any immediate or average balance between aggregate supply and demand, since any imbalance between the two is covered by term $UCINV_t$. However, if we *do* assume that there is some fairly rapid process which makes supply gravitate around demand, then on average $UCINV_t = 0$, and the regulating average level of net output becomes

$$Y_T = (Im_T + Iv_T + If_T) + C_T \tag{6}$$

All of the terms in the above expression represent *average* levels of the variables previously defined in period t and now defined over some longer period of time T appropriate to the slow adjustment process. Equation 6 can then be read as the familiar statement that when supply and demand balance on average, net output is the sum of total investment (in parentheses) and total consumption. Moreover, all schools of thought note that this total investment is composed of distinct components. For instance,

Quesnay distinguishes between *annual* advances (circulating capital) and the *original* advances (fixed capital), while Smith, Ricardo, and Marx distinguish between additional expenditures for wages and materials (investment in circulating capital) and those for fixed capital (Eltis, 1984, pp. 62, 75, 224; Marx, *Capital*, vol. II, ch. 21). Similarly, Keynes divides total investment into investment in 'fixed, working capital or liquid (i.e. inventory) capital' (Keynes, 1936, ch. 7, p. 75), Kalecki into 'fixed capital investment' and 'investment in [materials and final goods] inventories' (Kalecki, 1954, p. 106–8), Harrod into 'circulating and fixed capital' (Harrod, 1948, pp. 17–18), Hicks into 'fixed' and "working capital" (Hicks, 1965, ch. x, p. 105), and Robinson into investment in 'capital goods, including equipment, work-in-progress, technically necessary stocks of materials. etc.' (Robinson, 1966, p. 65).

Although all schools *note* the difference between circulating and fixed investment, the Classical/Marxian treatment of circulating capital differs in one crucial way from that of the Keynes/Kalecki tradition, in that the former links the purchase of additional inputs to the subsequent production of additional output.[5] This Classical/Marx/Leontief *input-output* link makes a crucial difference to dynamic analysis, because it tells us that while investment in fixed capital and inventories adds to capacity, investment in materials adds to output, so that any analysis of the dynamics of capacity utilization (i.e. of the relation of capacity to output) must pay close attention to the *difference* between the effects of these two components. Note that we are concerned here with the effects, and not the determinants, of these elements of investment.

Let us consider this point in more detail. Net investment in fixed capital If_T represents the change in the stock of plant and equipment. Its effect is to therefore *change aggregate capacity*. Abstracting from technical change, we can follow Marx and Harrod in assuming a constant fixed capital-capacity ratio $n = Kf/N$, where Kf_{T+1} = the stock of fixed capital and N_{T+1} = normal capacity net output, both at the beginning of period t 1 (end of period t). Then

$$N_{T+1} \quad N_T = (1/n)\,(Kf_{T+1} - Kf_T) = (1/n)\,If_T \tag{7}$$

Investment Iv_t in final goods inventories is somewhat different, in that it represents the *desired* change in final goods inventories, which will not equal the actual change unless the undesired change $UCINV_t = 0$ (aggregate supply = aggregate demand). Since we are indeed assuming the latter to hold over the average fast oscillation, Iv_t will equal the actual change in final goods inventories V.

$$V_{T+1} \quad V_T = Iv_t \tag{8}$$

level of capacity utilization will fall back toward normal, rather than spiralling ever upward as in the Harrodian case. Instead of the knife-edge, we have the Classical slow adjustment process.

Although circulating investment is mentioned in most theoretical analyses of growth, it is striking that it disappears from the empirical measures of output in orthodox national accounts. This is basically because modern accounts adopt the convention of treating current purchases of materials M_T as the production costs of current total output Q_T, just as they assume that depreciation of the capital stock equals retirements. Treating the materials component of current production in this way is tantamount to assuming away the production process, since it implicitly assumes a zero time of production. At a theoretical level, the same effect is achieved by substituting M_T for the term M_{T-1} in the definition of net output. This eliminates input investment *Zm* from equation 6 and the input investment share *am* from equation 11, which immediately leads to an internal inconsistency in the Harrodian formulation of the problem of dynamics. With input investment *Zm* (and its correlate Iv) eliminated, equation 6 reduces to the familiar Harrodian equation Y = *If/s,* where *If* is exogeneous to the short run, and where *Y* is now a *stationary* level of output. But then it is logically inconsistent to also use the same expression to define a warranted growth path or any dynamics around it. On the other hand, if we assume that total investment I includes both circulating and fixed investment with the former proportional to the latter, as Hicks does (Hicks, 1985, ch. 11, pp. 108-11) and Harrod implicitly does (Harrod, 1939, pp. 17–18), then we disable the capacity utilization adjustment mechanism. The inevitable consequence in either case is that the slow adjustment then becomes a runaway process.[7] This is precisely what we call the knife-edge.

3 STABILITY AROUND THE WARRANTED RATE OF GROWTH

In this section, we will demonstrate that once net output is correctly specified, the warranted rate is indeed stable. Assume that the desired inventory/sales ratio is constant. Since sales equal output over the average fast cycle, and since the input-output coefficient is assumed to be constant, desired inventory levels are proportional to material inputs so that desired investment in inventories will be proportional to materials investment. Formalizing this and substituting into equation 11 yields

$$Iv_T = vIm_T \text{ (v = desired inventory/materials ratio)} \tag{12}$$

$$am_T = (s - af_T)/(1 + v) \tag{13}$$

Next, define capacity utilization u_T as the ratio of actual net output to normal capacity.

$$u_T = Y_T/N_T \tag{14}$$

Now consider the determinants of the fixed investment share ak_t. As we noted earlier, in a dynamic environment all variables have trend paths, all targets are moving targets, and all adjustments in targets have to be adjustments relative to trend. Thus when capacity utilization is above normal, firms will be stimulated to raise investment in fixed capital faster than output and hence raise the fixed investment share ak_t, other things being equal. We can formalize this by assuming an investment reaction function in which the rate of change of the fixed investment share af_T is proportional to the degree of over- or underutilization of capacity $u_T - 1$.[8]

$$(af_{T+1} - af_T)/af_T = h(u_T - 1)$$
$$af_{T+1} = af_T + haf_T(u_T - 1) \tag{15}$$

All that remains is to relate the changes in capacity utilization u_T to changes in af_T and u_T. From equation 14

$$u_{T+1} = Y_{T+1}/N_{T+1} = (Y_T/N_T)(Y_{T+1}/Y_T)/(N_{T+1}/N_T)$$

Using equations 7 to get $N_{T+1}/N_T = 1 + (1/n)(If_T/Y_T)(Y_T/N_T) = 1 + af_T u_T/n$ and equation 9 to get $Y_{T+1}/Y_T = 1 + (1/m)(Im_T/Y_T) = 1 + am_T/m$, using equation 13 to substitute for am_T, and defining $m' = m(1 + v)$,

$$\begin{aligned} u_{T+1} &= u_T(1 + am_T/m)/(1 + af_T u_T/n) = u_T(1 + s/m' - af_T/m')/(1 + af_T u_T/n) \\ &= (n/m')[u_T(m' + s) - af_T u_T]/(n + af_T u_T) \\ &= (n/m')[u_T(m' + s) + n - (n + af_T u_T)]/(n + af_T u_T) \\ u_{T+1} &= A[(Bu_T + n)/(af_T u_T + n) - 1] \\ &\text{where } A = n/m',\ m' = (1 + v),\ B = m' + s \end{aligned} \tag{16}$$

Equations 15 and 16 form a first-order nonlinear (difference equation) dynamical system. For the range of values of the reaction coefficient *h* in which the system is stable, the growth rate converges to the warranted rate of growth and the level of capacity utilization converges to $u = 1$, for any single departure from this balance path. Moreover, when subject to

We come now to the least familiar component, Im_T. At the most abstract level, this too is produces a change in a stock – namely in the stock of materials. But there is a difference here. The stock of final goods is aimed at sales, the level of which depends more on buyers than on the firm. Similarly, although net investment in fixed capital builds up capacity, the use of this capacity over its (long and uncertain) lifetime depends on many factors which the firm does not control. But the purchase of materials is another matter. Firms purchase materials in order to produce commodities with them, and this particular decision is largely under the control of the firm. Thus purchases of materials tends to be strongly **linked to** corresponding flows of output. *This is exactly the assumption underlying Leontief's input-output analysis.* since the observed input-output coefficient are the ratios of purchased inputs to produced outputs. The well-known empirical stability of these ratios is evidence of the strong link between input purchase and output flow. Thus if the input-output coefficient $m = M_{T-1}/Y_T$ is constant, then the effect of investment in materials is to expand output in the subsequent period.

$$Y^{T+1} - Y_T = (1/m)\,(M, \quad M_{T-1}) = (1/m)\, Im_T \tag{9}$$

Substituting equations 7-9 into equation 6

$$Y_T = m(Y_{T+1} \quad Y_T) + (V_{T+1} - V_T) + n(N_{T+1} - N_T) + C_T \tag{10}$$

The presence of input investment $Im_T = m(Y_{T+1} \quad Y_T)$ on the right hand side of equation 10 is crucial for two reasons. First of all, it tells us that whereas fixed **investment** expands the capacity to produce the output and inventory investment expands the capacity to sell it, materials investment expands the output itself. Secondly, it shows us that the short run balance between aggregate supply and demand generally defines *a dynamic path, not merely a particular level,* **of** *net output Y.*

The above argument implies that any specification of short run output which fails to link input investment with output change implicitly assumes that the corresponding level of regulating output is *constant.* Conversely, any analysis which concludes that 'short run' factors determine a level (as opposed to a path) of output implicitly assumes materials investment is zero. In either case, the constancy of output in the 'short run' artificially displaces the discussion of growth to the 'long run'. What is worse, it also sets the stage for the Harrodian conclusion that growth is unstable, precisely because the very stabilizing mechanism – which arises **from** the *differential effects* of circulating and fixed investments on output and capacity, respectively – has been excluded from the **start.**[6]

Hicks' treatment is a partial exception to the rule. He arrives at virtually the same equation as equation 6, albeit from a quite different route. Whereas equation 6 is derived from a consideration of the input-output effects of materials investment, Hicks focuses on the determinants of materials investment, which he ties to the expected change in output. Since short run equilibrium expected output is the same as actual output, materials investment ends up being linked to the future change in actual output. Hicks notes that this immediately implies that short run equilibrium determines an output path, not a level (Hicks, 1985, ch. 11, pp. 108–11). But instead of pursuing the difference between the effects of circulating and fixed investment, he imposes the additional restriction that the former type of investment is *proportional* to the latter, in order to pursue the properties of 'the Equilibrium Path of a Keynes-type model (p. **112). This assumption is then carried over to the 'Harrod-type** model' (pp. 118–19). With this step, a crucial stabilizing mechanism is lost to the analysis, and the Harrodian knife-edge emerges.

The stabilizing influence of materials investment can be easily shown. Assume that aggregate consumption is proportional to aggregate net output, C = cY, let s = l-c = the average 'savings' ratio (a constant), substitute into equation 6, and divide through by Y.

$$am, + av, + af_T = 1 \qquad c = s = \text{constant} \tag{11}$$

where $am = Im/Y$ = the share of materials investment
$av = Iv/Y$ = the share of inventories investment
$ak = If/Y$ = the share of fixed investment

In the above expression, *am* is the component which leads to a change in output, while *av* and *af* leads to changes in sales and production capacitics, respectively. We will assume that *av* is proportional to *am* (this is equivalent to a constant desired inventory/sales ratio, as we shall see later). Since the savings propensity s is constant, the sum of the investment propensities must be constant, so that a rise in one component must come at the expense of the others. And with this, we have the means for solving the Harrodian puzzle. Suppose that supply and demand balance around some level of Y which happens to define a level of capacity utilization above normal. This means that fixed investment must rise relative to the trend of output in order to raise the trend of capacity relative to the trend of **output.** In other words, businesses will adjust their moving capacity targets by gradually raising the fixed investment share *ak.* And as this occurs, *two* things will happen simultaneously: first, the rise in *ak* will accelerate the growth in capacity; second, the concomitant fall in the share of materials **investment** *am* (since inventory investment is proportional to *am*) will decelerate the growth of actual output. The net result is that the

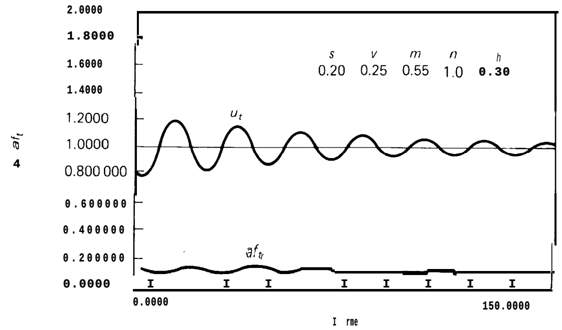

Figure 18.1 u_t and af_t (single shock)

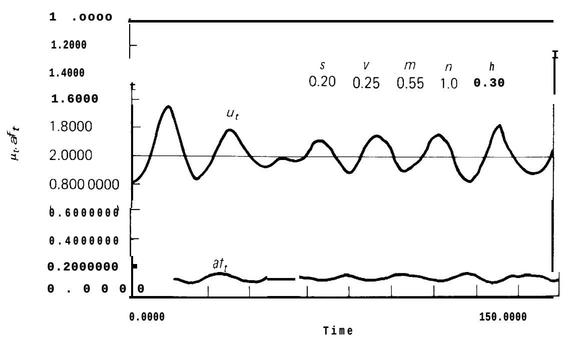

Figure 18.2 u_t and af_t with random shocks

random shocks representing the effects of the 'anarchy of capitalist production', the model ends up wandering around, but never settling down to, the warranted path. Figures 18.1-3 display the simulated behavior of the model in its stable range, and the Appendix analyzes its structure and stability.

The particular model developed above is only the simplest possible version of a general class of models which can be derived for alternate specifications of the fixed investment reaction function in equation 15. It is

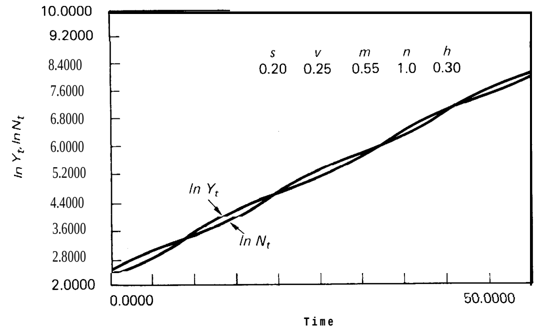

Figure 18.3 Output and capacity

interesting to note that if this very same function is specified in differential equation form, then the model is stable for *all* feasible (i.e. positive) values of the reaction coefficient *h*, and cyclically convergent for all plausible values (see Shaikh, 1989, appendix, part B). Alternate specifications can even yield limit cycles around the warranted path. What we get, therefore, is an integration of growth and cycle theory which is in the spirit of Kaldor and Harrod. The resulting picture of endogeneously generated turbulent growth is very much in the Classical and Marxian traditions.

Appendix

Equations 15-16 rewritten below constitute a nonlinear difference equation system with the Jacobian *J* shown below.

$$af_{T+1} = af_T(1-h) + haf_T u_T \tag{15}$$

$$uf_{T+1} = \frac{A(Bu_T + n)}{(af_T u_T + n)} - A \tag{16}$$

$$J = \begin{bmatrix} 1 - h + hu_T & haf_T \\ \dfrac{Au_T(Bu_T + n)}{(af_T u_T + n)^2} & \dfrac{AB}{(af_T u_T + n)} - \dfrac{Aaf_T(Bu_T + n)}{(af_T u_T + n)^2} \end{bmatrix}$$

Necessary and sufficient conditions for local stability (where TR = trace, DET = determinant, of the Jacobian evaluated at a critical point)[9] are:

(i) $1 - TR + DET > 0$
(ii) $1 + TR + DET > 0$
(iii) $1 - DET > 0$

Solving for $u_{T+1} = u_T = u$, and $af_{T+1} = af_T = af$ yields two critical points. The first point is $u = 0$, $af = 0$, in which case the Jacobian, Trace and Determinant reduce to

$$J_0 = \begin{bmatrix} 1-h & 0 \\ 0 & AB/n \end{bmatrix}, \quad TR_0 = 1 - h + AB/n, \quad DET_0 = (1-h)AB/n$$
where $A = n/m'$ and $m' = m(1+v)$

and this evidently unstable because condition i is not satisfied.

The second critical point is $u^* = 1$, $af^* = sA/(1+A)$. This represents the warranted path, because from equations 9, 13, and the above value of af^*, the growth rate of output is

$$g^*y = (Y^*_{T+1} - Y^*_T)/Y^*_T = (1/m)\ (I^*m_T/Y^*_T) = (1/m)\ am,$$
$$= (1/m)\ (s - af^*)/(1+v) = (1/m')\ (s - [sA/(1+A)])$$
$$= (s/m')/(1+A) = s/(n+m')$$

Here, v is the desired final goods inventory/materials ratio, m is the ratio of materials flow (and stock, since we have picked the time period equal to the period of production) to net sales, so that mv is the desired final goods inventory/sales ratio and $m' = m + mv$ is the ratio of materials and final goods inventories to net sales. But at the above critical point sales equals capacity because $u^* = 1$. Since n is the fixed capital/capacity ratio, $C = n + m'$ is the total capital/capacity ratio. Therefore

$$g^*y = s/C = \text{the warranted rate of growth}$$

For this critical point its Jacobian, TR and DET are

$$J_1 = \begin{bmatrix} 1 & haf^* \\ \dfrac{-(1+A)}{af^*+n} & \dfrac{n}{af^*+n} \end{bmatrix}, \quad TR_1 = 1 + n/(af^*+n), \quad DET_1 = \frac{h(1+A)af^*+n}{af^*+n}$$

It is easily verified that local stability conditions (i)-(ii) are satisfied for all positive values of h (since m, v, and n are all positive). But for the third condition we need $DET_1 < 1$, which requires that $h < h^* = 1/(1+A)$.

Notes

1. We may think of the slow adjustment process as operating on the *average* values of the fast adjustment variables (e.g., the propensity to invest in fixed capital is a function of the average level of capacity utilization over the fast oscillation).
2. Marx speaks of 'a cycle of lean and fat years' as the characteristic manner in which an average balance is achieved (*Capital*, vol. III, ch. XII, p. 208). See also Marx, 1970, p. 208.

3. Kalecki's theory typically partitions into short run, in which supply and demand are assumed to balance, medium run, in which he considers the business cycle around a stable level of output, and long run, in which he considers growth (Kalecki, 1959, ch. 14-15 and 1962, pp. 134-5). Keynes also saw 'growth [as] a long-period conception' (Kregel, 1980, p. 100), which led him to stumble over Harrod's notion of a steady advance as part of a growth environment (ibid., p. 99, fn. 5, and pp. 101-2).
4. By normal capacity output we mean the economically feasible capacity, which depends on costs, shiftwork, normal intensity and length of the working day, etc. This is quite different from engineering capacity, which is the technical upper limit to normal capacity. See Winston, 1974.
5. Eltis records that in Quesnay the 'agricultural output is proportional to the *annual* advances or circulating capital', so that additional advances result in additional output (Eltis, 1984, p. 75). Smith and Ricardo generalize this to all production, so that the level of output is tied to the prior expenditures for wages and materials (pp. 92, 224-5). This is why in the Classical tradition 'the rate of growth of circulating capital determines the [rate] of growth of output' (p. 93). Finally, Marx explicitly links the increase in circulating capital Ac + Av to subsequent increases in output and hence in surplus value As (Marx, *Capital*, vol. II, ch. 21).
6. This point is discussed in more detail in Shaikh, 1991. It is shown there that the Keynesian and Kaleckian short run equilibrium levels of output are static closures of the fast adjustment process.
7. See Shaikh, 1989B for a formal demonstration of this.
8. The term $u_t - 1 = (Y_t - N_t)/N_t$ can either be interpreted as a measure of over- or underutilization of capacity (depending on its sign), or it can be interpreted as a measure of over- or underfulfillment of expectations. In the latter case, we note that the capacity in place at the beginning of 'year' T is really the capacity planned for year T over the prior years in which the currently matured investment projects were initiated. But since investment projects can always be cancelled if it is clear that they are not needed, we could interpret current capacity N_t as an index of *last* year's expectation of this year's output. That is $N_t = (Y^e_t)_{t-1}$, where the t-1 subscript refers to the year in which the expectation was formed. On this basis, $u_t - 1$ is simply the per cent excess of actual over expected output.
9. Gandolfo, 1985, pp. 127 and 56.

References

Baumol, W. J. (1959), *Economic Dynamics* (New York: Macmillan).
Bleaney, M. (1976), *Underconsumption Theories* (New York: International Publishers).
Eltis, W. (1984), *The Classical Theory of Economic Growth* (New York: St Martin's Press).
Gandolfo, G. (1985), *Economic Dynamics: Methods and Models* (Amsterdam: North-Holland).
Garegnani, P. (1978), 'Notes on consumption, investment, and effective demand: a reply to Joan Robinson', *Cambridge Journal of Economics 3*, pp. 181-7.
Godley, W. and Cripps, F. (1983), *Macroeconomics* (Oxford: Oxford University Press).

Harrod, R. F. (1939), 'An essay in dynamic theory,' *Economic Journal* 49, March, 14-33.
——— (1948), *Towards a Dynamic Economics* (London: Macmillan).
Hicks, J. R. (1965), *Capital and Growth* (Oxford: Clarendon Press).
——— (1985), *Methods of Dynamic Economics* (Oxford: Clarendon Press).
Kaldor, N. (1960), *Essays on Value and Distribution* (London: Duckworth).
Kalecki, M. (1954), *The Theory of Economic Dynamics* (New York: Augustus M. Kelley).
(1962), 'Observations on the Theory of Growth', *Economic Journal,* March, pp. 135-53.
——— (1968), 'Trend and Business Cycle', in *Selected Essays on the Dynamics of the Capitalist Economy: 1933–1970,* (Cambridge: Cambridge University Press, 1971).
Keynes, J. M. (1936), *The General Theory of Employment, Interest, and Money* (New York: Harcourt, Brace, & World, 1964)
Kregel, J. A. (1980), 'Economic dynamics and the theory of steady growth: an historical essay on Harrod's "knife-edge",' *History of Political Economy,* 12, 1, 97-123.
Marx, K. 1970 [1865], *Wages, Prices and Profit,* reprinted in *Karl Marx and Frederick Engels: Selected Work in One Volume* (New York: International).
——— 1967 [1867], *Capital (New* York: International).
Robinson, J. (1966), *The Accumulation of Capital* (London: Macmillan).
Shaikh, A. (1984), 'The Transformation from Marx to Sraffa', in Ernest Mandel (ed.) *Ricardo, Marx, Sraffa*, (London: Verso).
——— (1989A), 'Accumulation, Finance and Effective Demand in Marx, Keynes, and Kalecki', in Willi Semmler (ed.) *Financial Dynamics and Business Cycles: New Perspectives, (New* York: M. E. Sharpe).
——— (1991), 'A Dynamic Approach to the Theory of Effective Demand', in D. Papadimitriou (ed.) *Profits and Instability* (London: Macmillan).
Winston, G. C. (1974),